CASE FILE COPY

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL REPORT
R-50

APPROXIMATIONS FOR THE THERMODYNAMIC AND TRANSPORT PROPERTIES OF HIGH-TEMPERATURE AIR

By C. FREDERICK HANSEN

1959

TECHNICAL REPORT R-50

APPROXIMATIONS FOR THE THERMODYNAMIC AND TRANSPORT PROPERTIES OF HIGH-TEMPERATURE AIR

By C. FREDERICK HANSEN

Ames Research Center
Moffett Field, Calif.

TECHNICAL REPORT R–50

APPROXIMATIONS FOR THE THERMODYNAMIC AND TRANSPORT PROPERTIES OF HIGH-TEMPERATURE AIR [1]

By C. Frederick Hansen

SUMMARY

The thermodynamic and transport properties of high-temperature air are found in closed form starting from approximate partition functions for the major components in air and neglecting all minor components. The compressibility, enthalpy, entropy, the specific heats, the speed of sound, the coefficients of viscosity and of thermal conductivity, and the Prandtl numbers for air are tabulated from 500° to 15,000° K over a range of pressure from 0.0001 to 100 atmospheres. The energy of air and the mol fractions of the major components of air can be found from the tabulated values for compressibility and enthalpy. It is predicted that the Prandtl number for fully ionized air, which is in complete equilibrium, will become small compared to unity, the order of 0.01, and this implies that boundary layers in such flow will be very transparent to heat flux.

INTRODUCTION

It is axiomatic that the science of aerodynamics must be based on a good understanding of the atmospheric medium through which vehicles are to fly. Under subsonic flight conditions, air may be treated as an ideal gas composed of rigid, rotating diatomic molecules. The thermodynamic properties of such a gas are well known and they are accounted for in the gas flow equations by the familiar ratio of specific heats, which in this case is a constant. Under supersonic flight conditions, air may be raised to temperatures where the molecules can no longer be treated as simple, rigid rotators. At relatively low supersonic speeds, vibrational energy is excited and then the specific heats become functions of temperature. However, both the thermodynamic

and transport properties of air in vibrational excitation can be predicted with fair accuracy by the methods of quantum statistics and kinetic theory (ref. 1), and the air-flow relations can be modified accordingly. Eggers (ref. 2) has calculated the effects of vibrational energy on the one-dimensional, inviscid flow of diatomic gases, for example.

Further changes in air properties may occur at still higher flight velocity. Flight velocities of practical interest have now increased from low supersonic speeds to the escape velocity, 37,000 feet per second, and higher. Vehicles which travel at these hypervelocities excite the air to such high temperatures that the molecules not only vibrate but may dissociate into atoms and even ionize. Under these conditions, the behavior of air deviates widely from that of an ideal gas and the thermodynamic and transport properties all become functions of pressure as well as of temperature. It is, of course, essential to evaluate these functions in order to calculate the pattern of air flow about high-speed vehicles, the viscous and pressure forces which result, and the heat flux which occurs between the air and the vehicle

The equilibrium thermodynamic properties of a gas can be calculated with good confidence, provided the energy levels of the gas particles and the degeneracy of these levels are known. For monatomic and diatomic gases this information can generally be deduced from spectroscopic data with such accuracy that the calculated thermodynamic properties can be trusted to very high temperatures, even where experimental confirmation is lacking. In the case of air, however, one of the important energy terms was not known with confidence until recently, namely, the dissociation

---

energy of molecular nitrogen. This uncertainty arose because the available spectroscopic data were consistent with two different models for nitrogen dissociation, one leading to a dissociation energy of 7.37 electron volts per molecule and the other to 9.76 electron volts per molecule. At first, the lower value was widely accepted as the most probable one (Herzberg, ref. 3). Krieger and White (ref. 4) and Hirschfelder and Curtiss (ref. 5) have published tables of thermodynamic properties of high-temperature air based on this value. Gaydon (ref. 6) was perhaps one of the first advocates of the view that the higher value was the correct one. Subsequently a number of experiments were performed which confirmed Gaydon's opinion, among them the measurements of strong shock waves in nitrogen made by Christian, Duff, and Yarger (ref. 7) and the detonation studies made by Kistiakowsky, Knight, and Malin (ref. 8). This rendered the work of references 4 and 5 obsolete, but shortly thereafter Gilmore (ref. 9) computed the chemical composition, energy, entropy, compressibility, and pressure of air as functions of temperature and density based on the higher value for the dissociation of nitrogen. Later, Hilsenrath and Beckett (ref. 10) published a similar table of these properties, but in much smaller increments of temperature and density. The calculations in both of these references (9 and 10) are highly refined in the sense that they not only account for the major components of air and their most significant energy states, but they also take into account a large number of the higher energy states which are infrequently excited, even at high temperatures, and most of the very minor chemical components of air are included. Therefore these works are among the most detailed estimates for the thermodynamic properties of air which have been made. It is not likely that they will become obsolete, since the values of all the important energy levels used in these calculations are now quite secure. Logan and Treanor (ref. 11) and Hochstim (ref. 12) have also prepared extensive tables of air properties including the specific heats and speed of sound. The calculations above (refs. 9 through 12) make use of multiple iteration procedures and for a variety of reasons it is desirable to have, in addition, approximate expressions in closed form which can be solved without iteration. Such solutions would be valuable, for example, used as a subroutine in

an electronic computing program or for preparing tables in small intervals suitable for the calculation of the flow of real air by the method of characteristics.

In contrast to the fairly satisfactory state of development in regard to thermodynamic properties, knowledge of the transport properties of air at high temperatures is in an elementary state. It is generally agreed that an accurate calculation of the transport properties should be based on the rather rigorous theory of Chapman and Enskog for monatomic gases (ref. 13). The extension of this theory given by Wang-Chang and Uhlenbeck (ref. 14) would be used for molecular gases with internal energy. For such calculations it is necessary to know the interaction potentials which exist between the gas particles so that the so-called collision integrals can be evaluated. Hirschfelder, Curtiss. and Bird (ref. 15) have developed methods of using intermolecular collision theory to the stage where the collision integrals and the transport properties of inert molecular gases can be calculated with good accuracy. However, when the air dissociates and ionizes, as it does at high temperatures, the atom-atom, atom-ion, and atom-molecule potentials are needed, and these are not sufficiently well known to calculate the collision integrals. At the present time, calculation of these potentials is being attempted by quantum mechanical methods, but it may be some time before reliable solutions are available. In any event, there is an urgent need for an estimate of the properties of air, due to the demands created by the expanding realm of very high-speed flight. Therefore, an engineering approximation for the transport properties of high-temperature air would be valuable in the interim while more exact solutions are being prepared. Even after more precise solutions are available, an approximation giving the transport properties in closed form may be just as desirable as it is for the thermodynamic proper ies of air.

In view of the needs outlined above, it is the purpose of this paper to develop approximate expressions for the properties of air over the range of temperatures and pressures encountered by vehicles traversing the atmosphere at speeds up to escape velocity. The principle which shall be used in deriving these expressions is that they shall be made as simple in form as possible, without appreciable sacrifice of reliability relative to the

accuracy inherent in the present state of theoretical development. For the thermodynamic properties this is accomplished by keeping only those terms which are necessary to yield a final result within a few percent of the more exact solutions.

In making estimates of the transport properties, one is faced with two alternatives. The transport coefficients may be calculated by the fairly rigorous but complex method of Chapman and Enskog, as noted above, or by the simple but approximate kinetic theory of hard elastic spheres (ref. 1). If good estimates of the interparticle potentials were available, the choice would immediately fall on the former method, of course. However, at present, the interparticle potentials are sufficiently uncertain so that it is doubtful whether the potentially greater accuracy of the more rigorous method could be realized. It has therefore been decided to defer the labor of making the rigorous calculations until the necessary potentials are available, and to use the simple kinetic theory for the present. It may be noted that this simple approach need not be discounted unduly, for it gives answers in the low temperature range that are adequate for many engineering uses; the more rigorous methods are needed for corrections which are only the order of 10 percent or less. Moreover, beside the ease of calculation, the elementary kinetic theory possesses some advantage in providing a simplified physical insight into the problem.

The properties of air which will be evaluated are the compressibility (i.e., the correction to the ideal gas equation of state), the energy, enthalpy, entropy, specific heat at constant pressure and at constant density, the speed of sound, the viscosity, thermal conductivity, and the Prandtl number. All of these properties will be evaluated for conditions of complete thermal equilibrium. Because of the finite reaction rates, these quantities will need to be modified for processes where changes in state are rapid compared to the rate of approach to chemical equilibrium. Nonequilibrium effects will probably be encountered in very high-altitude flight because the approach to equilibrium is slow at the low pressures experienced there. However, there is experimental evidence that equilibrium is essentially realized in flow of dissociating air under conditions encountered in flight at moderate altitudes and speeds (ref. 16). In addition, Hirsch-

felder (ref. 17) has argued that heat transfer in pure conduction processes will correspond to equilibrium values if the reaction rate in one direction is rapid. Therefore, the thermodynamic and transport properties of air which are based on equilibrium conditions should be a convenient reference for the nonequilibrium values, and apply directly to some practical problems as well.

SYMBOLS

a	speed of sound (zero frequency)
a_i, b_i	stoichiometric coefficients for components A_i and B_i
A_i, B_i	components of a chemical reaction
C	Sutherland's constant (eq. (58))
C_i	specific heat per mol at constant density for component i
C_p	specific heat per mol at constant pressure
C_p'	partial specific heat per mol at constant pressure, $\sum_i x_i(C_i + R)$
C_v	specific heat per mol at constant density
D	dissociation energy per molecule, also diffusion coefficient
D_{ij}	binary diffusion coefficient for molecules of type i into molecules of type j
e	base of natural logarithms, also electron charge
e^-	electron
E	energy per mol, also electric field strength
E_i	energy per mol of component i
E_0	energy per mol at zero absolute temperature
g_i	degeneracy of the ith state
g_n	degeneracy of the nth electronic state
h	Planck's constant
H	enthalpy per mol
H_i	enthalpy per mol of component i
I	molecular moment of inertia, also ionization energy per molecule
I_0	resonance potential for ionization
J	rotational quantum number
k	Boltzmann constant, also thermal conductivity
k_0	reference coefficient of thermal conductivity (eq. (77))

k_n, k'	partial coefficient of thermal conductivity due to molecular collisions
k_r	partial coefficient of thermal conductivity due to chemical reaction
K_c	chemical equilibrium constant for concentration units
K_p	chemical equilibrium constant for pressure units
Le'	partial Lewis number, $\dfrac{D\rho C_p'}{Mk'}$
ln	logarithm to the base e
m	mass of a gas particle
M_i	weight per mol of component i
M	mean weight per mol of a gas mixture
M_0	weight per mol for undissociated air molecules
n	vibrational quantum number, also electronic quantum number, also concentration in moles per unit volume
$n(A_i),$ $n(B_i), \ldots$	concentration of components $A_i,$ $B_i, \ldots$ in moles per unit volume
N	nitrogen atom, also atoms in general
N_0	Avagadro number, molecules per mol
N^+	nitrogen plus ion, also plus ions in general
N_2	nitrogen molecule
NO	nitric oxide
O	oxygen atom
O^+	oxygen plus ion
O_2	oxygen molecule
p	pressure
p_0	reference pressure, 1 atmosphere
$p(A_i),$ $p(B_i), \ldots$	partial pressure of components $A_i,$ $B_i, \ldots$
Pr	Prandtl number, $\dfrac{C_p \eta}{Mk}$
Pr'	partial Prandtl number, $\dfrac{C_p' \eta}{Mk'}$
Q	total partition function
Q_t	translational partition function
Q_r	rotational partition function
Q_v	vibrational partition function
Q_e	electronic partition function
Q_c	total partition function for a standard state of unit concentration, $\dfrac{p}{RT} Q$
Q_p	total partition function for a standard state of unit pressure, pQ

$Q_p(A_i)$ $Q_p(B_i) \ldots$	total partition functions for components $A_i, B_i, \ldots$
r	distance between atoms
r_e	distance between atoms where the potential is a minimum
R	universal gas constant, energy per mol deg
S	entropy per mol
S_i	entropy per mol of component i at 1 atmosphere pressure
S_0	collision cross section for undissociated air molecules
S_{ij} or $S(i\text{-}j)$	collision cross section for particle i with particle j
T	absolute temperature
u_i	mean molecular velocity for molecule type i
u_0	mean molecular velocity for undissociated air molecules
U	potential energy between gas particles
x	mol fraction
x_i	mol fraction of component i
$x(A_i) \ldots$	mol fraction of component $A_i \ldots$
Z	compressibility, $\dfrac{pM_0}{\rho RT}$ or $\dfrac{M_0}{M}$
α	molecular symmetry number (equal 2 for homonuclear diatomic molecules), also polarizability
β	Morse function constant (eq. (61))
γ	ratio of specific heats, $\dfrac{C_p}{C_v}$
ϵ	fraction of molecules which are dissociated or of atoms which are ionized
ϵ_i	energy of the ith state
ϵ_n	energy of the nth electronic state
ζ	dimensionless distance parameter, $\dfrac{r}{r_e} - 1$
η	coefficient of viscosity
η_0	reference coefficient of viscosity (eq. (67))
λ_i	mean free path for molecule type i
λ_0	reference mean free path (eq. (71))
ν	vibrational frequency
ρ_i	density of molecule type i
ρ_0	reference density (eq. (70))
σ	collision diameter

SUBSCRIPTS

p	partial derivative at constant pressure
ρ	partial derivative at constant density

s	partial derivative at constant entropy
i, j	indices referring to molecules type i and j
t, r, v, e	indices referring to the contribution of translational, rotational, vibrational, and electronic energy modes, respectively
1	oxygen dissociation reaction
2	nitrogen dissociation reaction
3	atom ionization reactions

THERMODYNAMIC PROPERTIES

As a preliminary, a brief review of some of the results of statistical mechanics will be given. This will include the definitions of the partition functions and will summarize those relations between these functions and the thermodynamic properties of gases which will be used in the approximations to follow.

PARTITION FUNCTIONS

All of the thermodynamic properties of a gas may be calculated from its partition function. Consequently, the first step in determining the properties of air is to calculate the partition functions for the components in air. The partition function may be defined as

$$Q=\sum_i g_i e^{-\frac{\epsilon_i}{kT}} \tag{1}$$

where ϵ_i is the energy of the ith state of the gas particle and g_i is the degeneracy, that is, the number of states of the particle which have this same energy level. The energy may be due to the translational, rotational, or vibrational motion of the particle, or to the motion of the electrons within the particle. The temperatures being considered in this paper are in all cases low enough that the excited nuclear energy states may be disregarded. The usual assumption is made that no coupling exists between the different modes of energy. Then the partition function may be expressed as the product

$$Q=Q_t Q_r Q_v Q_e \tag{2a}$$

The factors on the right side of equation (2a) are, respectively, the partition functions associated with the translational, rotational, vibrational, and electronic energy levels of the gas particle. Each factor is determined independently by an equation of the same form as equation (1).

By the methods of statistical mechanics it is found that for diatomic molecules these factors are:

$$Q_t=\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\frac{RT}{p} \tag{2b}$$

$$Q_r=\sum_{J=0}^{\infty}(2J+1)e^{-\frac{h^2 J(J+1)}{8\pi^2 IkT}}\simeq\frac{8\pi^2 IkT}{\alpha h^2} \tag{2c}$$

$$Q_v=\sum_{n=0}^{\infty}e^{-\frac{nh\nu}{kT}}=\left(1-e^{-\frac{h\nu}{kT}}\right)^{-1} \tag{2d}$$

$$Q_e=\sum_{n=0}^{\infty}g_n e^{-\frac{\epsilon_n}{kT}} \tag{2e}$$

The notations used above are common ones and the derivation of the equations may be found in any standard text on statistical mechanics, such as reference 18 or 19. For monatomic particles, which have no modes of rotational or vibrational energy, the rotational and vibrational partition functions take the value unity. The translational and electronic partition functions for such particles take the same form as equations (2b) and (2e), respectively.

Consistent with the stated purpose of this paper, only those exponential terms are included in the electronic partition function (eq. (2e)) for which the energy levels, ϵ_n, are less than six times kT at the maximum temperature considered (15,000° K). Actually, the levels are so widely split in this range that the closest to this cutoff is the sixth state of the atomic nitrogen ion, just a little more than four times kT at 15,000° K.

Table I presents the atomic and molecular constants which were used in calculating the partition functions. The molecular constants for rotation, vibration, dissociation energy, and electronic energy levels were taken from Herzberg (ref. 3). The 9.76 electron-volt value for nitrogen dissociation is used, and it is assumed that the rotational and vibrational constants for all excited electronic states are the same as for the ground state. The atomic energy levels are taken from Moore (ref. 20). The constants have been rounded off to the nearest 0.5 percent, and the second and third electronic energy levels of atomic nitrogen have been combined since they lie within 0.5 percent of each other. The same treatment applies to the second and third electronic levels of the atomic oxygen positive ion.

The functions which are to be used directly in the calculations to follow are the logarithms of the partition functions. From the partition functions constants (table I) and equations (2a) through (2e), these functions become

$$\ln Q(N_2) = \frac{7}{2} \ln T - 0.42 - \ln\left(1 - e^{-\frac{3390}{T}}\right) - \ln p \tag{3a}$$

$$\ln Q(O_2) = \frac{7}{2} \ln T + 0.11 - \ln\left(1 - e^{-\frac{2270}{T}}\right)$$
$$+ \ln\left(3 + 2e^{-\frac{11390}{T}} + e^{-\frac{18990}{T}}\right) - \ln p \tag{3b}$$

$$\ln Q(O) = \frac{5}{2} \ln T + 0.50 + \ln\left(5 + 3e^{-\frac{228}{T}} + e^{-\frac{326}{T}} + 5e^{-\frac{22800}{T}} + e^{-\frac{48600}{T}}\right) - \ln p \tag{3c}$$

$$\ln Q(N) = \frac{5}{2} \ln T + 0.30 + \ln\left(4 + 10e^{-\frac{27700}{T}} + 6e^{-\frac{41500}{T}}\right) - \ln p \tag{3d}$$

$$\ln Q(O^+) = \frac{5}{2} \ln T + 0.50 + \ln\left(4 + 10^{-\frac{38600}{T}} + 6e^{-\frac{58200}{T}}\right) - \ln p \tag{3e}$$

$$\ln Q(N^+) = \frac{5}{2} \ln T + 0.30 + \ln\left(1 + 3e^{-\frac{70.6}{T}} + 5e^{-\frac{188.9}{T}} + 5e^{-\frac{22000}{T}} + e^{-\frac{47000}{T}} + 5e^{-\frac{67900}{T}}\right) - \ln p \tag{3f}$$

$$\ln Q(e^-) = \frac{5}{2} \ln T - 14.24 - \ln p \tag{3g}$$

where T is the temperature in degrees Kelvin and p is given in atmospheres.

It will be surmised that those components of air which are not represented by partition functions above are to be neglected in the approximations which follow. The absence of a partition function for nitric oxide may be found surprising, but it will be seen later that, to the order of accuracy being considered here, the formation of nitric oxide may be neglected in computing thermodynamic properties over a wide range of pressure and temperature conditions, including those conditions which will generally be encountered in high-speed flight through the atmosphere. This occurs because NO has about the same thermodynamic properties as an average for nitrogen and oxygen molecules, and the NO formation does not greatly influence the equilibrium mol fractions of molecules and of atoms. The possible influence of nitric oxide on the transport properties of air will be considered later.

ENERGY, ENTHALPY, AND SPECIFIC HEATS FOR THE COMPONENTS OF AIR

According to statistical mechanics, the energy and enthalpy per mol of pure gas are given by the following relations:

$$\frac{E - E_0}{RT} = T\left(\frac{\partial \ln Q}{\partial T}\right)_\rho = T\frac{d \ln Q_c}{dT} \tag{4}$$

$$\frac{H - E_0}{RT} = T\left(\frac{\partial \ln Q}{\partial T}\right)_p = T\frac{d \ln Q_p}{dT} \tag{5}$$

The quantities Q_c and Q_p are the partition functions for the standard states of unit concentration and of unit pressure, respectively. These are related to the total partition function by

$$Q_c = \frac{p}{RT} Q \tag{6a}$$

$$Q_p = pQ \tag{6b}$$

and they are functions only of temperature so that it is their total derivatives which are related to the energy and enthalpy as given in equations (4) and (5). The quantity E_0 is a constant representing the energy of the gas at zero absolute temperature. The choice of this level is arbitrary, but by convention E_0 is taken as zero for the molecules of nitrogen and oxygen. Then E_0 for the neutral atoms is just one half the energy of dissociation per mol of diatomic molecules, and E_0 for the ionized atoms is the sum of this dissociation energy and the energy of ionization. With all the ionization energy attributed to the ionized atoms, E_0 for the electrons must be taken as zero, of course.

By definition the specific heats per mol of pure gas are

$$C_v = \left(\frac{\partial E}{\partial T}\right)_\rho \tag{7}$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p \tag{8}$$

From equations (2a), (2b), (2c), (4), and (5) it is seen that the energy and enthalpy per mol of gas due to translation and electronic excitation are given by

$$\left(\frac{E-E_0}{RT}\right)_{t+e}=\frac{3}{2}+\frac{\sum \frac{\epsilon_n}{kT}g_n e^{-\frac{\epsilon_n}{kT}}}{\sum g_n e^{-\frac{\epsilon_n}{kT}}} \qquad (9)$$

$$\left(\frac{H-E_0}{RT}\right)_{t+e}=\left(\frac{E-E_0}{RT}\right)_{t+e}+1 \qquad (10)$$

and the specific heats given by equations (7) and (8) can be expressed

$$\left(\frac{C_v}{R}\right)_{t+e}=\frac{3}{2}+\frac{\sum\left(\frac{\epsilon_n}{kT}\right)^2 g_n e^{-\frac{\epsilon_n}{kT}}}{\sum g_n e^{-\frac{\epsilon_n}{kT}}}-\left(\frac{\sum \frac{\epsilon_n}{kT}g_n e^{-\frac{\epsilon_n}{kT}}}{\sum g_n e^{-\frac{\epsilon_n}{kT}}}\right)^2 \qquad (11)$$

$$\left(\frac{C_p}{R}\right)_{t+e}=\left(\frac{C_v}{R}\right)_{t+e}+1 \qquad (12)$$

Equations (9), (10), (11), and (12) give that part of the energy, enthalpy, and the specific heats which is due to the translational and electronic energy for either atoms or molecules. The contributions of rotational and vibrational energy must also be included for the molecular case, of course. According to equations (2c) and (2d) the expression

$$\left(\frac{E}{RT}\right)_{r+v}=1+\frac{h\nu}{kT}\left(e^{\frac{h\nu}{kT}}-1\right)^{-1} \qquad (13)$$

should be added to equations (9) and (10) in order to obtain the total energy and enthalpy for diatomic molecules, while

$$\left(\frac{C}{R}\right)_{r+v}=1+\left(\frac{h\nu}{2kT}\right)^2\left(\sinh \frac{h\nu}{2kT}\right)^{-2} \qquad (14)$$

should be added to equations (11) and (12) to get the total specific heats for these particles.

The entropy of a pure gas is related to its partition function by

$$\frac{S}{R}=\ln Q+T\left(\frac{\partial \ln Q}{\partial T}\right)_p \qquad (15)$$

and from equations (5) and (6b), the entropy of the gas at unit pressure may be expressed

$$\frac{S}{R}=\ln Q_p+\frac{H-E_0}{RT} \qquad (16)$$

The calculated values of the partition functions for a standard state of 1 atmosphere pressure, the enthalpy, and the specific heat at constant pressure are given in table II in 500° K increments of temperature for the major components of air. The concentration standardized partition functions, the energy, entropy, and the specific heat at constant density are not listed since they may be easily found from the properties tabulated and equations (6), (10), (12), and (16).

EQUILIBRIUM CONSTANTS

In order to determine the equilibrium mol fractions for the components of air, it will be necessary to calculate the equilibrium constants for the chemical reactions which occur. These chemical reactions may be expressed in the generalized form

$$\sum a_i A_i \rightleftarrows \sum b_i B_i \qquad (17)$$
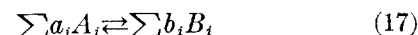
where the A_i are the reactants, the B_i the products, and a_i and b_i are their respective stoichiometric coefficients. The pressure equilibrium constant for this reaction is defined in terms of the partial pressures

$$K_p=\frac{\Pi p^{b_i}(B_i)}{\Pi p^{a_i}(A_i)} \qquad (18)$$

and it is related to the partition functions by (see ref. 18)

$$\ln K_p=-\frac{\Delta E_0}{RT}+\sum b_i \ln Q_p(B_i)-\sum a_i \ln Q_p(A_i) \qquad (19)$$

where

$$\Delta E_0 \equiv \sum b_i E_0(B_i)-\sum a_i E_0(A_i)$$

is the zero point energy of the products less the zero point energy of the reactants, both referred to their standard states. The reactions considered here are the dissociation of molecular oxygen and of molecular nitrogen and the ionization of atomic oxygen and of atomic nitrogen. According to equation (19) and the constants given in table I, the equilibrium constants for these reactions are

$$\ln K_p(O_2 \rightarrow 2O) = -\frac{59,000}{T} + 2\ln Q_p(O) - \ln Q_p(O_2) \tag{20a}$$

$$\ln K_p(N_2 \rightarrow 2N) = -\frac{113,200}{T} + 2\ln Q_p(N) - \ln Q_p(N_2) \tag{20b}$$

$$\ln K_p(O \rightarrow O^+ + e^-) = -\frac{158,000}{T} + \ln Q_p(O^+) + \ln Q_p(e^-) - \ln Q_p(O) \tag{20c}$$

$$\ln K_p(N \rightarrow N^+ + e^-) = -\frac{168,800}{T} + \ln Q_p(N^+) + \ln Q_p(e^-) - \ln Q_p(N) \tag{20d}$$

The concentration equilibrium constant is defined by

$$K_c = \frac{\Pi n^{b_i}(B_i)}{\Pi n^{a_i}(A_i)} \tag{21}$$

where $n(A_i)$ and $n(B_i)$ are, respectively, the concentrations of the chemical reactants and products. This quantity will also be needed for subsequent calculations, and it is obtained by replacing the pressure standardized partition functions, Q_p, with the corresponding concentration standardized partition functions, Q_c, in equation (19). From equation (6) it is seen that

$$K_c = K_p(RT)^{\Sigma a_i - \Sigma b_i} \tag{22}$$

The logarithmic derivatives of the equilibrium constants will also be required later. From equations (5), (10), and (19) these become

$$T\frac{d\ln K_c}{dT} = \frac{\Delta E_0}{RT} + \sum b_i \left(\frac{E-E_0}{RT}\right)_{B_i}$$

$$- \sum a_i \left(\frac{E-E_0}{RT}\right)_{A_i} \tag{23}$$

$$T\frac{d\ln K_p}{dT} = T\frac{d\ln K_c}{dT} + \sum b_i - \sum a_i \tag{24}$$

The equilibrium constants and their logarithmic derivatives for the reactions represented by equations (20a) through (20d) are listed as functions of temperature in table III. A population weighted average quantity is given for the oxygen and nitrogen ionization reaction. These quantities will now be used in calculating the component mol fractions and their derivatives.

CALCULATION OF THE EQUILIBRIUM MOL FRACTIONS AND THEIR DERIVATIVES

The possibility that approximate solutions in closed form could be obtained for the properties

of air suggests itself upon examination of the results of Gilmore (ref. 9). His tables of the composition of air show that there are four chemical reactions of major importance. These are the dissociation of molecular oxygen and molecular nitrogen, and the ionization of atomic oxygen and of atomic nitrogen.

$$O_2 \rightarrow 2O \tag{25a}$$

$$N_2 \rightarrow 2N \tag{25b}$$

$$O \rightarrow O^+ + e^- \tag{25c}$$

$$N \rightarrow N^+ + e^- \tag{25d}$$
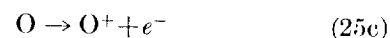
With one exception, all other reactions which occur yield component concentrations which are the order of 0.1 percent, or less. The exception is the formation of nitric oxide, NO, which at sea level density may become as much as 10 percent of the air around 5000° K. However, even this much nitric oxide does not strongly influence the resulting thermodynamic properties of air, and at densities less than 0.01 normal sea level density, where the NO is less than 1 percent at its maximum, the effects are very small.

Two distinctive features of the chemical reactions given above are observable from Gilmore's results. The first is that at all pressures the dissociation of oxygen is essentially complete before the dissociation of nitrogen begins. This means that these two reactions can be treated independently for the purposes of approximation. The second feature is that nitrogen and oxygen atoms ionize at about the same temperature and with about the same energy changes. Consequently, it is possible to assume that once air is completely dissociated, all atoms constitute a single species which has the population weighted average properties of the nitrogen and oxygen atoms.

The equation of state will be defined

$$\frac{p}{\rho}=\frac{ZRT}{M_0} \tag{26}$$

where Z is the compressibility. To the approximation that all of the particles obey the ideal gas law, Z represents the total number of mols formed from a mol of initially undissociated air. It is also equal to the ratio of the initial molecular weight of undissociated air to the mean molecular weight, $M_0/\overline{M}$. If ϵ_1 is the fraction of molecules which dissociate into oxygen atoms, ϵ_2 the fraction of molecules which dissociate into nitrogen atoms, and ϵ_3 the fraction of atoms which are ionized, then the compressibility is given by

$$Z=1+\epsilon_1+\epsilon_2+2\epsilon_3 \tag{27}$$

The reactions are now assumed to be independent and, in view of the order of approximation being considered, the ratio of nitrogen to oxygen has simply been taken as 4 to 1. Then at relatively low temperatures only three major components exist simultaneously: molecular nitrogen, molecular oxygen, and atomic oxygen. The partial pressures for these three components may be expressed

$$p(N_2)=x(N_2)p=\frac{0.8}{1+\epsilon_1}\,p \tag{28a}$$

$$p(O_2)=x(O_2)p=\frac{0.2-\epsilon_1}{1+\epsilon_1}\,p \tag{28b}$$

$$p(O)=x(O)p=\frac{2\epsilon_1}{1+\epsilon_1}\,p \tag{28c}$$

The equilibrium constant for the oxygen dissociation reaction (eq. (20a)) is

$$K_{p1}=\frac{p^2(O)}{p(O_2)}=\frac{4\epsilon_1^2p}{(1+\epsilon_1)(0.2-\epsilon_1)} \tag{29}$$

Then the fraction ϵ_1 is found by solving the quadratic equation (29)

$$\epsilon_1=\frac{-0.8+\sqrt{0.64+0.8\left(1+\frac{4p}{K_{p1}}\right)}}{2\left(1+\frac{4p}{K_{p1}}\right)} \tag{30}$$

The condition of no dissociation which occurs at low temperatures is, of course, just the limit where ϵ_1 is zero.

At intermediate temperatures ϵ_1 approaches the limit 0.2, the oxygen is completely dissociated, and the nitrogen diocssiation commences. For this case, molecular nitrogen, atomic nitrogen, and atomic oxygen coexist and the partial pressures of the components are

$$p(N_2)=x(N_2)p=\frac{0.8-\epsilon_2}{1.2+\epsilon_2}p \tag{31a}$$

$$p(N)=x(N)p=\frac{2\epsilon_2}{1.2+\epsilon_2}\,p \tag{31b}$$

$$p(O)=x(O)p=\frac{0.4}{1.2+\epsilon_2}\,p \tag{31c}$$

The equilibrium constant for the nitrogen dissociation reaction (eq. (20b)) is

$$K_{p2}=\frac{p^2(N)}{p(N_2)}=\frac{4\epsilon_2^2p}{(1.2+\epsilon_2)(0.8-\epsilon_2)} \tag{32}$$

whence ϵ_2 is given by

$$\epsilon_2=\frac{-0.4+\sqrt{0.16+3.84\left(1+\frac{4p}{K_{p2}}\right)}}{2\left(1+\frac{4p}{K_{p2}}\right)} \tag{33}$$

At high temperatures, ϵ_2 approaches the limit 0.8, the dissociation of nitrogen is also complete, and the ionization of the atoms begins. For this case we assume that the atoms are a single homogeneous species symbolized by N, and then the partial pressures become

$$p(N)=x(N)p=\frac{1-\epsilon_3}{1+\epsilon_3}p \tag{34a}$$

$$p(N^+)=x(N^+)p=\frac{\epsilon_3}{1+\epsilon_3}p \tag{34b}$$

$$p(e^-)=x(e^-)p=\frac{\epsilon_3}{1+\epsilon_3}p \tag{34c}$$

The ionization reaction equilibrium constant is

$$K_{p3}=\frac{p(N^+)p(e^-)}{p(N)}=\frac{\epsilon_3^2p}{1-\epsilon_3^2} \tag{35}$$

and then ϵ_3 becomes

$$\epsilon_3=\left(1+\frac{p}{K_{p3}}\right)^{-\frac{1}{2}} \tag{36}$$

The equilibrium constant K_{p3} is taken to be a population-weighted average of the constants for

the oxygen and nitrogen ionization reactions (eqs. (20c) and (20d)). That is,

$$K_{p3}=0.2\ K_p(O\to O^+ + e^-) + 0.8\ K_p(N\to N^+ + e^-)$$

(37)

The component mol fractions in air are then given by

$$x(O_2)=\frac{0.2-\epsilon_1}{Z}$$

(38)

$$x(N_2)=\frac{0.8-\epsilon_2}{Z}$$

(39)

$$x(O)=\frac{2\epsilon_1-0.4\epsilon_3}{Z}$$

(40)

$$x(N)=\frac{2\epsilon_2-1.6\epsilon_3}{Z}$$

(41)

$$x(N^+ + O^+)=x(e^-)=\frac{2\epsilon_3}{Z}$$

(42)

The derivatives of the products Zx_i will be needed in addition to the mol fractions and from equations (38) through (42) it is seen that these are proportional to the derivatives $\partial\epsilon_1/\partial T$, $\partial\epsilon_2/\partial T$, and $\partial\epsilon_3/\partial T$. From equations (28) and (29) the partial derivative of ϵ_1 at constant pressure is

$$\left(\frac{\partial\epsilon_1}{\partial T}\right)_p=\frac{d\ln K_{p1}/dT}{2/\epsilon_1-1/(1+\epsilon_1)+1/(0.2-\epsilon_1)}$$

(43a)

Similarly, the partial derivatives of ϵ_2 and ϵ_3 at constant pressure are

$$\left(\frac{\partial\epsilon_2}{dT}\right)_p=\frac{d\ln K_{p2}/dT}{2/\epsilon_2-1/(1.2+\epsilon_2)+1/(0.8-\epsilon_2)}$$

(43b)

$$\left(\frac{\partial\epsilon_3}{dT}\right)_p=\frac{d\ln K_{p3}/dT}{2/\epsilon_3-1/(1+\epsilon_3)+1/(1-\epsilon_3)}$$

(43c)

In order to find the same partial derivatives at constant density, it is convenient to use the equilibrium constant in concentration units. For example,

$$K_{c1}=\frac{4\epsilon_1^2 n}{(1+\epsilon_1)\ (0.2-\epsilon_1)}=\frac{4\epsilon_1^2}{(0.2-\epsilon_1)}\frac{\rho}{M_0}$$

(44)

whence the partial derivative of ϵ_1 at constant density is

$$\left(\frac{\partial\epsilon_1}{dT}\right)_\rho=\frac{d\ln K_{c1}/dT}{2/\epsilon_1+1/(0.2-\epsilon_1)}$$

(45a)

in a similar way, the partial derivatives of ϵ_2 and ϵ_3 at constant density are

$$\left(\frac{\partial\epsilon_2}{dT}\right)_\rho=\frac{d\ln K_{c2}/dT}{2/\epsilon_2+1/(0.8-\epsilon_2)}$$

(45b)

$$\left(\frac{\partial\epsilon_3}{\partial T}\right)_\rho=\frac{d\ln K_{c3}/dT}{2/\epsilon_3+1/(1-\epsilon_3)}$$

(45c)

The preceding relations are given in terms of temperature and pressure as the independent variables. This is a convenient form in which to use the solutions in the analysis of nearly constant pressure processes such as occur in boundary-layer flow or at the stagnation region of high-speed vehicles. However, the precise calculations of air properties have been carried out as constant density functions of temperature (refs. 9 and 10) and, in order to make a comparison with these calculations, it is desirable to have the approximate solutions given in the same variables. This is very simply done by expressing the dissociated and ionized fractions in terms of density and the equilibrium constants in concentration units. Then (see eq. (44))

$$\epsilon_1=\frac{-(K_{c1}M_0/\rho)+\sqrt{(K_{c1}M_0/\rho)^2+3.2(K_{c1}M_0/\rho)}}{2}$$

(46a)

In a similar way

$$\epsilon_2=\frac{-(K_{c2}M_0/\rho)+\sqrt{(K_{c2}M_0/\rho)^2+12.8(K_{c2}M_0/\rho)}}{8}$$

(46b)

$$\epsilon_3=\frac{-(K_{c3}M_0/\rho)+\sqrt{(K_{c3}M_0/\rho)^2+8(K_{c3}M_0/\rho)}}{4}$$

(46c)

All the other calculations follow as before, only the quantities ϵ_1, ϵ_2, and ϵ_3 from equations (46a), (46b), and (46c) replace those from equations (30), (33), and (36).

With the preceding relations in hand, we are in position to calculate the energy, entropy, specific heat, and speed of sound for air.

ENERGY, SPECIFIC HEAT, ENTROPY, AND SPEED OF SOUND FOR AIR IN EQUILIBRIUM

The energy per mol of air is simply the sum

$$E=\sum_i x_i E_i$$

(47)
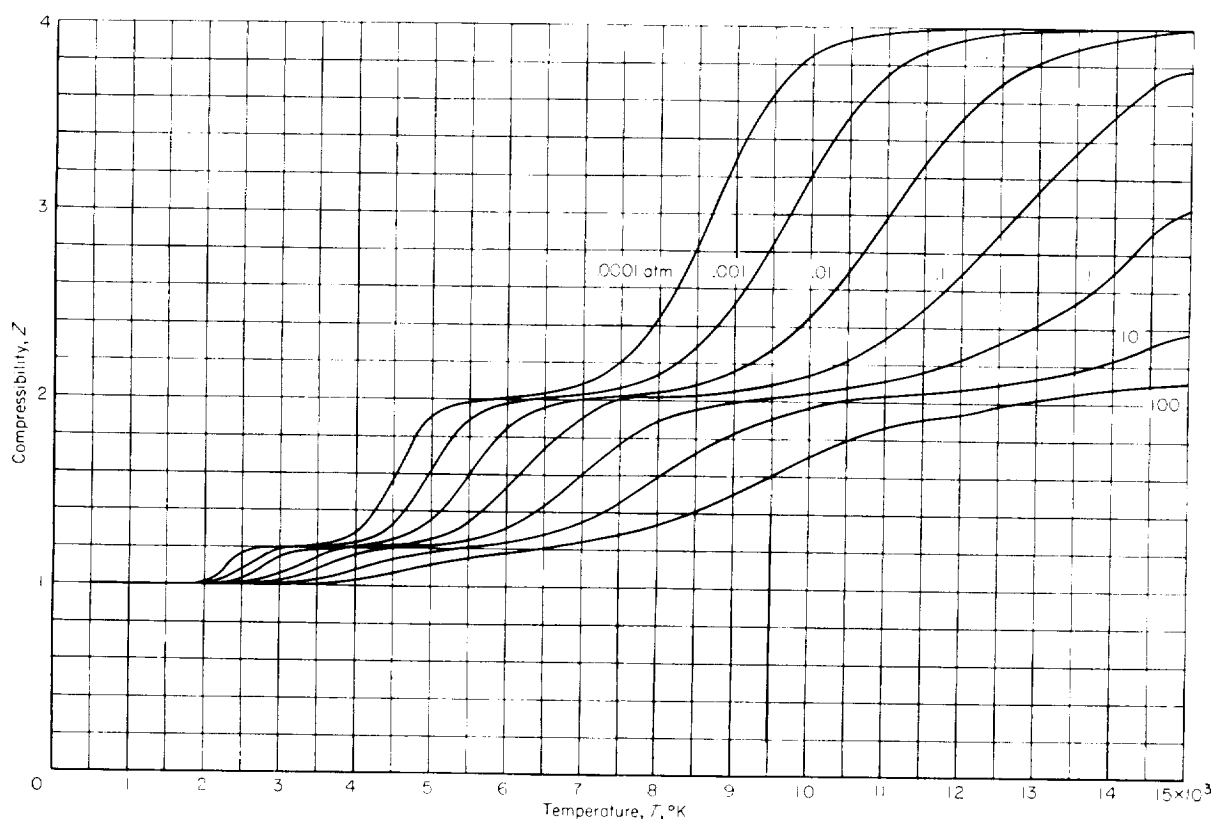
FIGURE 1.—Compressibility of air as a function of temperature.

where E_i is the energy per mol for component i. Generally in aerodynamic problems the energy per fixed mass of gas is needed rather than the energy per mol. The mass of gas in a mol of undissociated air (29 gm) will be used as the constant reference quantity for this purpose. Then, in dimensionless form, the energy per mol of initially undissociated air is

$$\frac{ZE}{RT}=Z\sum_i x_i \frac{E_i}{RT} \qquad (48)$$

while the dimensionless enthalpy per initial mol of air becomes

$$\frac{ZH}{RT}=\frac{ZE}{RT}+Z \qquad (49)$$

The compressibility, Z, and the dimensionless enthalpy, ZH/RT, which have been calculated from the preceding equations, are listed in tables IV(a) and IV(b). The energy is easily obtained from these values via equation (49). Compressibility and energy are graphed as functions of temperature in figures 1 and 2. The entropy per

initial mol of air is simply obtained from the entropies of the components of air through the relation

$$\frac{ZS}{R}=Z\left(\sum_i x_i \frac{S_i}{R}-\sum x_i \ln x_i-\ln\frac{p}{p_0}\right) \qquad (50)$$

where p_0 is the reference pressure for the standard state, in this case 1 atmosphere. The entropy values are listed in table IV(c) and are shown graphically in figure 3.

At this point it is desirable to compare the foregoing approximate solutions with the precise calculations made by Hilsenrath and Beckett (ref. 10). Figure 4(a) shows the percentage deviation in the approximate solutions for compressibility at densities of 10^2, 10^0, 10^{-2}, 10^{-4}, and 10^{-6} Amagat (unit density at STP). Figures 4(b) and 4(c) show, respectively, the deviations in the approximate solutions for enthalpy and entropy for the same densities. It is found that the errors are generally less than 2 percent at densities between 1 and 10^{-6} Amagat. It is to be expected that the solutions lose accuracy at
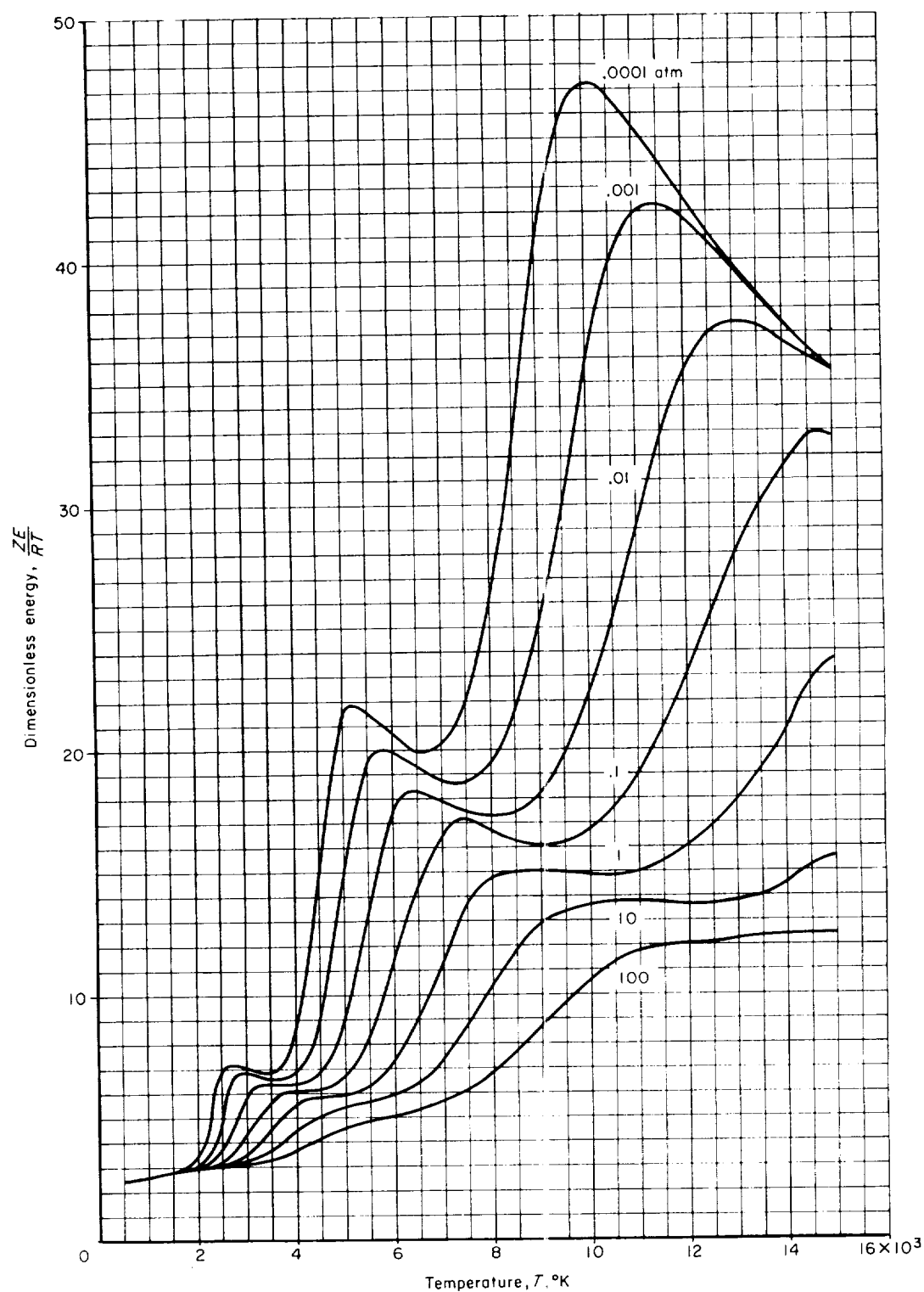
FIGURE 2.—Energy of air as a function of temperature.

(a) Compressibility.
(b) Enthalpy.
(c) Entropy.
(d) Ratio of specific heats.
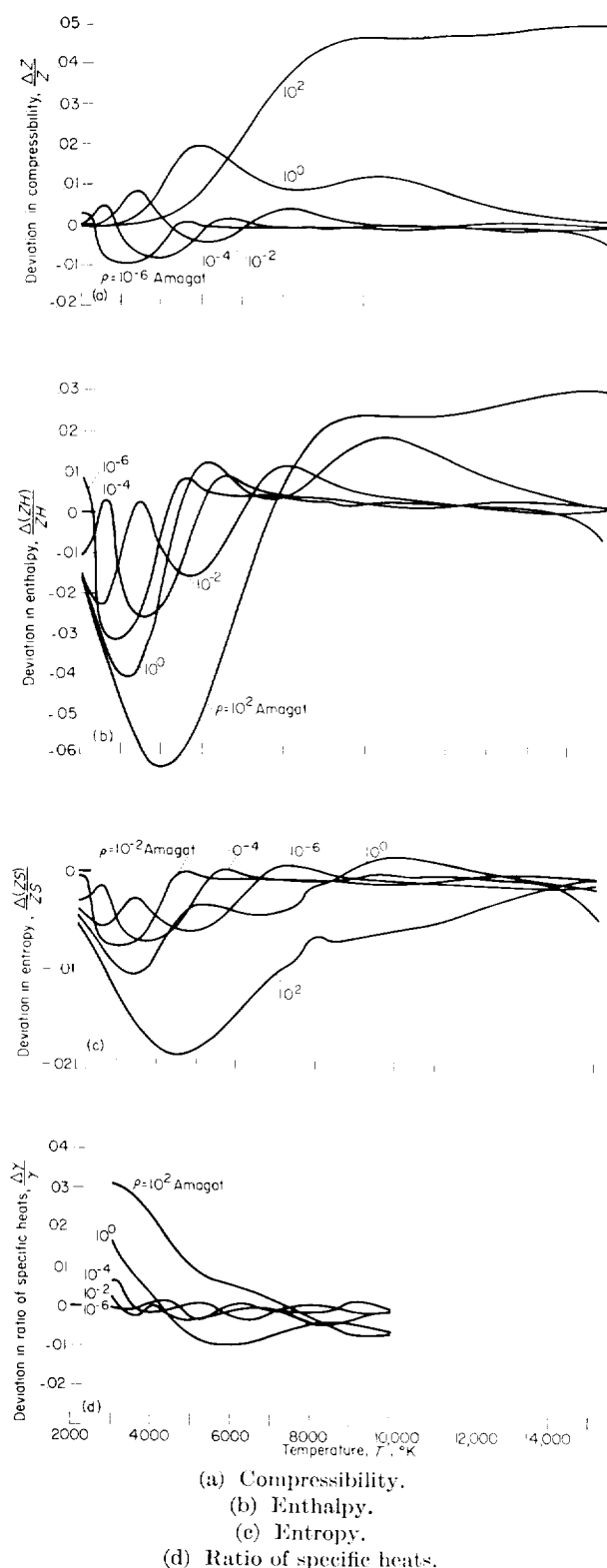
FIGURE 4.—Deviations in the approximate solutions for the thermodynamic properties of air.

very high densities since some of the neglected minor reactions begin to take on increasing importance, especially the formation of NO at intermediate temperatures. Moreover, the major reactions begin to occur simultaneously so that it is no longer so good an assumption that they are independent of one another. Even so the errors are only about 6 percent or less up to 100 Amagats density, and this may still be acceptable accuracy for many purposes. In general, the accuracy of the approximations increases as density decreases, and the solutions may be extended to densities even lower than 10^{-6} Amagat, if desired.

It is necessary to take the derivatives of the products ZE and ZH for specific heats if the latter are to have their usual meanings, that is, the change in energy which occurs in a fixed mass of gas per degree temperature change. Thus the constant density specific heat per initial mol of air is given by

$$\frac{ZC_v}{R}=\frac{1}{R}\left(\frac{\partial ZE}{\partial T}\right)_\rho=Z\sum_i x_i\frac{C_i}{R}+T\sum_i\left(\frac{E_i}{RT}\right)\left(\frac{\partial Zx_i}{\partial T}\right)_\rho \tag{51}$$

where C_i is the derivative of energy for component i, that is, dE_i/dT. The corresponding specific heat for constant pressure is given by

$$\frac{ZC_p}{R}=\frac{1}{R}\left(\frac{\partial ZH}{\partial T}\right)_p=Z\sum_i x_i\left(\frac{C_i}{R}+1\right)$$
$$+T\sum_i\left(\frac{E_i}{RT}+1\right)\left(\frac{\partial Zx_i}{\partial T}\right)_p \tag{52}$$

The specific heat at constant pressure is listed in table IV(d) and the ratio of specific heats, γ, is given in table IV(e). It may be noted that when chemical reactions occur, the difference between the specific heats per mol is not equal to the gas constant, as in an ideal gas, but is given by

$$\left(\frac{C_p}{R}-\frac{C_v}{R}\right)=1+\frac{T}{Z}\sum_i\left[\frac{H_i}{RT}\left(\frac{\partial Zx_i}{\partial T}\right)_p-\frac{E_i}{RT}\left(\frac{\partial Zx_i}{\partial T}\right)_\rho\right] \tag{53}$$

The approximations are not expected to be as accurate for the specific heats as for the other thermodynamic properties since one higher order derivative of the partition functions is involved. However, the results are generally within 10 percent of the values calculated by Logan and
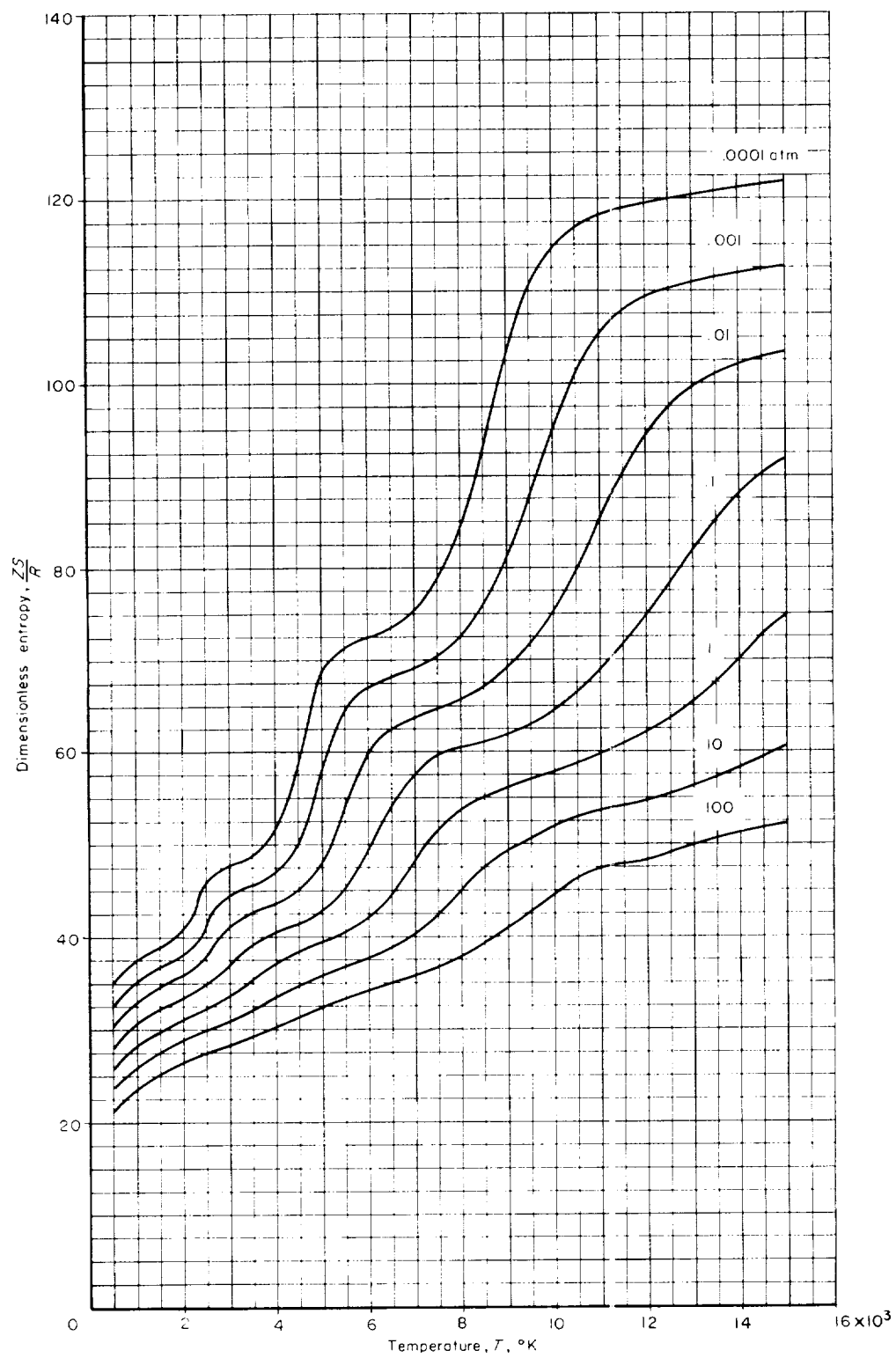
FIGURE 3.—Entropy of air as a function of temperature.
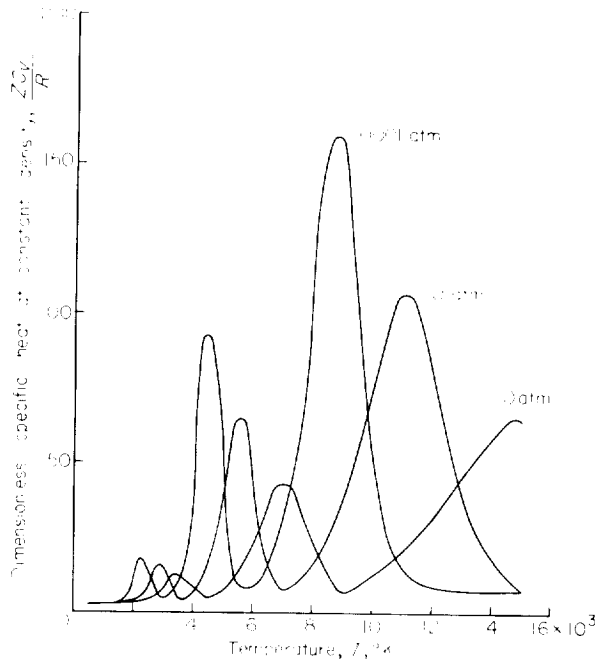
FIGURE 5.—Specific heat of air at constant density as a function of temperature.

Treanor (ref. 11). Fortunately, one does not usually need to use the specific heats directly in aerodynamic calculations, anyway, but rather the ratio of specific heats, γ. The deviations in γ from the values given in reference 11 are shown in figure 4(d), and it is seen that the present approximations preserve the accuracy of this quantity within a few percent.

The chemical reactions have a marked effect on the specific heat functions. For example, figure 5 shows the specific heat at constant density, C_v, as a function of temperature for pressures of 0.0001, 0.01, and 1 atmosphere. At low temperature C_v increases from $5R/2$ to $7R/2$ as the vibrational modes of energy become excited. Then, with increasing temperature, the specific heat goes through three distinct maxima where the chemical components change most rapidly with temperature; the first maximum is due to the oxygen dissociation reaction, the second to the nitrogen dissociation, and the third to the ionization reactions. When pressure decreases, these maxima increase in sharpness and in magnitude as they shift to lower temperatures.

The foregoing specific heat values enable one to

calculate the speed of sound in air. This speed of sound will be defined by

$$a^2=\left(\frac{\partial p}{\partial \rho}\right)_s \qquad (54)$$

that is, the limiting value as the frequency of sound approaches zero. The partial derivative given in equation (54) will not be calculated directly, since it is not convenient to treat the entropy as an independent variable which can be held constant (see eq. (50)). However, equation (54) may be transformed with perfect generality into the form

$$a^2=\gamma\left(\frac{\partial p}{\partial \rho}\right)_T \qquad (55)$$

where γ is the ratio of the specific heats, C_p/C_v. Equation (55) is, in turn, equivalent to

$$a^2=-\gamma\frac{(\partial p/\partial T)_\rho}{(\partial \rho/\partial T)_p} \qquad (56)$$

which, from the equation of state (eq. (26)), may be expressed in terms of variables which have already been calculated

$$\frac{a^2\rho}{p}=\gamma\frac{1-(T/Z)(\partial Z/\partial T)_\rho}{1+(T/Z)(\partial Z/\partial T)_p} \qquad (57)$$

The dimensionless speed of sound parameter, $a^2\rho/p$, is listed in table IV(f) and is plotted as a function of temperature in figure 6. The second term on the right side of equation (57) is generally near unity, so that figure 6 is also indicative of the variation in γ with temperature.

AERODYNAMIC CONSIDERATIONS

The thermodynamic properties obtained at this point are those required to perform calculations of inviscid air-flow problems. These properties are given for a range of temperature from 500° to 15,000° K and of pressure from 0.0001 to 100 atmospheres. It is of interest now to examine the altitude and velocity at which these conditions will occur in flight. A grid of the pressure and temperature at the stagnation point of a body in flight is shown in figure 7 as a function of flight altitude and velocity. The stagnation enthalpy per unit mass was simply taken as one half the velocity squared, and the stagnation pressure was related to the static pressure (and thus to altitude)
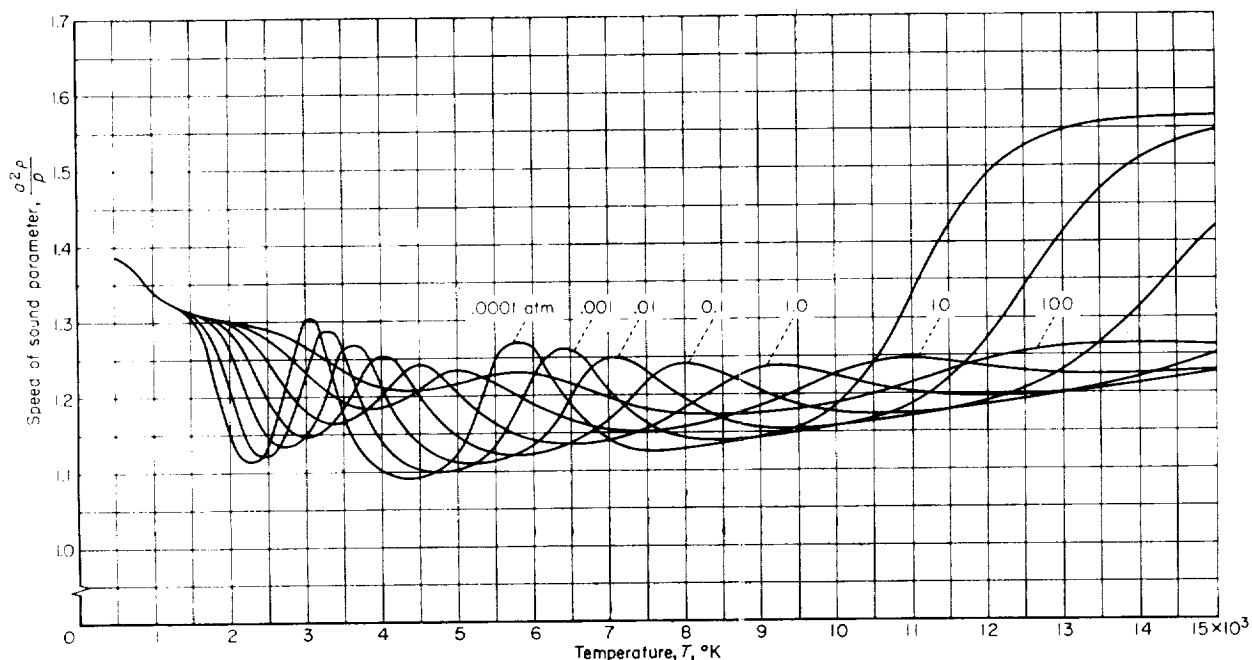
FIGURE 6.—Speed of sound parameter for air as a function of temperature.

with the results of Feldman (ref. 21), who computed the pressure ratio developed across normal shock waves in air at various altitudes. Generally lower temperatures and pressures will be attained at regions other than the stagnation region, so the range of variables will be adequate for such

FIGURE 7.—Stagnation temperature and pressure in air as a function of altitude and velocity.

cases also. It can be seen that the thermodynamic properties of air can be approximated in closed form over the range of conditions of current interest in aerodynamics. Some of the results of these approximations will next be used in estimating the transport properties of high-temperature air.

TRANSPORT PROPERTIES

COLLISION CROSS SECTIONS

Consider first some qualitative aspects of the collisions between gas particles. The particle trajectories are influenced by a potential U which is negligible at long range, which may be attractive or repulsive at intermediate range, but which always becomes strongly repulsive at very short range. A particle with kinetic energy kT will not be greatly deflected if it passes only through that part of the potential field where $|U| << kT$. On the other hand, the particle will be deflected appreciably if it passes where $|U| >> kT$. The direction of the deflection is unimportant so far as transport properties are concerned, for it is the absolute value of the deflection angle which determines the change in mass, momentum, and energy fluxes caused by the collision. To a first approximation, the absolute value of this deflection is independent of the sign of the potential and the

effective collision diameter σ is the order of the largest distance where $U=\pm kT$. The effective collision cross section S will be defined as $\pi\sigma^2$.

In the rigorous treatment of the transport properties of gases, the effective collision cross section S is found to be an integral of the deflection angles produced by collisions and this integral is a function of the relative velocity of the colliding particles. The so-called "collision integral" is a function of temperature only which is S times a velocity function integrated over all velocities. Thus the collision integral may be thought of as a weighted average collision cross section, and the transport coefficients can be related directly to these integrals. However, not all the interparticle potentials have been developed which are needed to calculate the collision integrals for air. In the present paper then, plausible estimates of the effective collision cross sections will be used to determine the mean free paths for hard elastic spheres, and for such particles the transport coefficients can be related to these mean free paths (ref. 1). The effects of the interaction potentials will be taken into account by letting the spherical cross sections be appropriate functions of temperature.

The effective cross sections for collisions between diatomic molecules can be obtained quite accurately by the collision integral method. However, at high temperatures, the very steep repulsive portion of the intermolecular potential is penetrated so that the molecules behave essentially like hard spheres. Consistent with the approximations which follow, it will be sufficient to use the Sutherland formula for the molecular cross section S_0.

$$\frac{S_0}{S_\infty}=1+\frac{C}{T} \tag{58}$$

where the constant C is 112° K and S_∞, the cross section at infinite temperature, is $3.14\times10^{-15}\text{cm}^2$, for the case of air molecules.

For atomic collisions, the picture is complicated by the fact that two atoms may approach each other along any one of a number of potentials. For example, the potentials between two normal nitrogen atoms are shown qualitatively in sketch (a) (see ref. 3). The lowest lying of these potentials, designated U_0, has the lowest total electron spin and it is the one normally responsible for the vibrational energy levels observed in the stable molecular state. Therefore U_0 can be expressed

quantitatively from experimental spectroscopic data. Unfortunately, the higher lying potentials for the atoms in air are not known quantitatively at present, so we are forced to estimate an average collision cross section for all of the potentials by means of the known lowest lying potential. For this purpose it is assumed that the collision diameters σ are given by

$$U_0(\sigma)=-kT \tag{59}$$

and these diameters will be used to evaluate the coefficients of momentum and energy transfer. A somewhat deeper penetration of the potential is normally required for a collision to affect the particle flux, so that the diameters σ' which will be used to evaluate the diffusion coefficients are assumed to be given by

$$U_0(\sigma')=-2kT \tag{60}$$

It may be pointed out that Hirschfelder and Eliason (ref. 22) have examined the relation between values of the transport coefficients given by the hard sphere model and by the more rigorous collision integral method. They find that $U(\sigma)$ and $U(\sigma')$ are about $-0.6kT$ for a wide variety of attractive potentials, that $U(\sigma)$ is about $0.9kT$ and $U(\sigma')$ is about $1.6kT$ for a similar variety of repulsive potentials. If all the interparticle potentials were known, it would be simple to use these criteria to obtain a weighted average
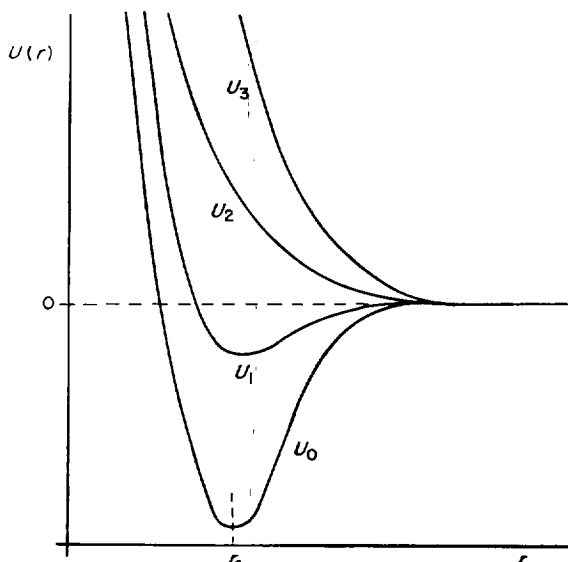
SKETCH (a).

collision diameter. All the potentials are known for two normal hydrogen atoms (ref. 23) and it is found for this case that equations (59) and (60) yield values for σ and σ' which agree with the weighted average collision diameters within 8 percent over the range of temperatures from 1000° to 15,000° K. Of course, there is no assurance that these same relations will hold as closely for collisions between the atoms in air. In fact the average collision diameters for normal oxygen and nitrogen atoms will probably be overestimated by equations (59) and (60), since the shallow intermediate potentials (such as U_1 and U_2, sketch (a)) must be considered for these atoms, whereas they do not occur for hydrogen. The effect of these intermediate potentials will be partly compensated for by the fact that some of the atoms will be in excited electronic states which have collision diameters the order of three times larger than the normal atoms (ref. 22). The fraction of atoms in excited states is small over most of the temperature and pressure range considered so that collisions between excited particles are relatively rare. The encounters between an excited and a normal atom are the ones which significantly influence the mean free paths, and the cross sections for such collisions are about four times normal. The fraction of each constituent in air which is excited is listed as a function of temperature in the following table:

T, ° K	Fraction of oxygen atoms excited	Fraction of nitrogen atoms excited	Fraction of ionized atoms excited
4,000	0.002	0.002	
6,000	.013	.025	0.012
8,000	.032	.081	.032
10,000	.055	.153	.060
12,000	.079	.228	.089
14,000			.120

The first two excited states of atomic oxygen and of the atomic nitrogen plus ion are very close to the ground state (see table I) and so they have not been counted as excited states. It can be seen that the fraction of excited particles in air will generally be less than 10 percent and in the very worst case considered about 20 percent of the atoms will be excited. This occurs at 100 atmospheres pressure where the ionization reaction is repressed until temperatures beyond 12,000° K are reached and 23 percent of the nitrogen and 8 percent of the oxygen atoms are excited. For a pure gas in which excited particles have three

times normal collision diameters, the average mean free path is decreased 28 percent when 10 percent of the particles are excited, and is decreased 45 percent when 20 percent of the particles are excited. Thus the error introduced by neglecting the long-range forces of the excited atoms is probably the same order of magnitude and opposite in effect to the uncertainties caused by neglecting the shortened range of the shallow, intermediate collision potentials.

The lowest lying atom-atom potentials may be approximated by the Morse function

$$U_0 = D[(1 - e^{-\beta \zeta})^2 - 1] \qquad (61)$$

where D is the dissociation energy and ζ is the dimensionless distance parameter $(r/r_e - 1)$; r_e is the interatomic distance at which U_0 is a minimum (sketch (a)); β is a constant related to the vibrational frequency ν of the stable molecule and is given by $\nu(2\pi^2 I/D)^{1/2}$. Although the Morse function is not very accurate at long ranges, it will be needed only at rather high temperatures where the potential is fairly well described for $|U_0| \geq kT$. The Morse function constants were taken from Herzberg (ref. 3) and these are listed in the following table:

Atomic particles	D/k, ° K	β	r_e, angstroms
O–O	59,000	3.24	1.207
N–N	113,200	2.96	1.094
N–O	75,400	3.18	1.151
O–O⁺	75,200	3.18	1.123
N–N⁺	101,200	2.94	1.116

From equations (59) and (61), the momentum and energy transfer collision diameter σ is given by

$$\frac{\sigma}{r_e} = 1 - \beta^{-1} \ln \left(1 - \sqrt{1 - \frac{kT}{D}} \right) \qquad (62a)$$

and by equations (60) and (61) the diffusion collision diameter σ' is given by

$$\frac{\sigma'}{r_e} = 1 - \beta^{-1} \ln \left(1 - \sqrt{1 - \frac{2kT}{D}} \right) \qquad (62b)$$

The atom-molecule collision diameters will be taken as the arithmetic average of the atom-atom and of the molecule-molecule collision diameters.

This assumption corresponds to the concept that the collision diameter is a measure of the effective range of the electron distribution about the molecule or atom, and that a collision occurs whenever these electron distributions overlap.

With the onset of ionization, several additional types of collisions must be considered. These are the ion-atom, electron-atom, and the ion-ion, ion-electron, or electron-electron collisions. The ion-atom collisions may be treated in the same manner as the atom-atom collisions, since the spectroscopic data for ionized molecules are available to be used in setting up the Morse potential function between the ion and the neutral atom. The Morse function constants for these cases, which occur in air, are also listed in the preceding table. The ion-ion, ion-electron, and electron-electron collision diameters all depend on long range electrostatic forces of identical magnitude, and the collision cross section for such particles is

$$S(e-e) = \pi \left(\frac{e^2}{kT}\right)^2 \ln \Lambda \qquad (63)$$

The first factor $\pi(e^2/kT)^2$ is the cross section for an encounter between two isolated charged particles. This cross section is based on the same criterion for the diameter as used above, namely $|U(\sigma)| = kT$, where the function U is just the coulomb potential between two charges. The second factor, $\ln \Lambda$, is a correction designed to account for the charge shielding effect on the many long-range collisions which occur in a plasma of finite density. According to reference 24, this correction is given by

$$\ln \Lambda = \ln \frac{3}{2e^3} \left(\frac{k^3 T^3}{\pi n_e N_0}\right)^{1/2} \qquad (63a)$$

where n_e is the mol concentration of all charged particles, both positive and negative. Actually at the densities considered here, the effects of this correction are not very large until the gas is almost completely ionized.

For electron-atom collisions, the atom will be polarized by the approach of the electron and there will result a charge-dipole type interaction. In order to calculate the magnitude of this interaction, the polarizability of the atom, α, will be needed. The polarizability is defined as the dipole moment produced in the atom by an electric field, E, of unit strength. The interaction energy of the induced dipole moment with the field is $\alpha E^2/2$, and when the field due to the presence of the electron is substituted for E, this energy becomes

$$U = -\frac{e^2 \alpha}{2r^4} \qquad (64)$$

It will be noted that the polarizability need not be known with great precision, since the collision diameter depends only on the fourth root of this quantity. Experimental values of polarizability are only given for the molecular state of oxygen and nitrogen. However, Joos (ref. 25) gives an approximate method of calculating the polarizability of atoms in alternating fields which in the limit for steady state fields becomes

$$\alpha = \frac{1}{m}\left(\frac{he}{2\pi I_0}\right)^2 \qquad (65)$$

where m in this case is the mass of an electron and I_0 is the resonance potential for ionization of the atom. This resonance potential is 9.11 e.v. for oxygen atoms and 10.28 e.v. for nitrogen atoms (ref. 26). Using these values, one finds the polarizability is $13.2 \times 10^{-25} \mathrm{cm}^3$ and 10.3×10^{-25} cm^3 for oxygen and nitrogen atoms, respectively.

The collision diameters will enter the calculations to follow in the form of a ratio with the collision diameter for two diatomic molecules at the same temperature, S_0. These ratios were calculated by the methods outlined above for those collisions which occur between the major components in high-temperature air. The results are given in table V as functions of temperature. Wherever two or more types of collision give the same result within 5 percent, there is not much point to considering them separately, in view of the approximations involved. Therefore, for such cases, the values have been weighted and averaged, and only those final averages are given in table V. The atom-molecule cross sections $S(O_2-O)$, $S(N_2-N)$, and $S(N_2-O)$ are very close and these are all designated by the single symbol $S(N_2-N)$. Similarly the atom-atom and atom-ion cross sections $S(O-O)$, $S(N-N)$, $S(N-O)$, $S(N-N^+)$, and $S(O-O^+)$ are all grouped under the notation $S(N-N)$. The atom-electron cross sections $S(N-e)$ and $S(O-e)$ are given the single average value listed under $S(N-e)$. In the same way, all the atom-molecule and atom-atom diffusion cross sections are given by $S'(N_2-N)$ and $S'(N-N)$, respectively. The electron-atom cross sections

for diffusion are not differentiated from the cross sections for momentum and energy transport.

VISCOSITY

The viscosity of air will be calculated from the simple summation formula for a mixture of hard spherical molecules

$$\eta = \frac{5\pi}{32} \sum_i \rho_i u_i \lambda_i \tag{66}$$

In view of the order of approximations involved, a more sophisticated formulation than this does not seem warranted. The constant $5\pi/32$ is chosen to agree with the value which accounts for the persistence in velocity of the higher speed molecules (ref. 1). It will be convenient to use, as a reference value, the viscosity which air would have at the same temperature if the molecules did not dissociate or ionize

$$\eta_0 = \frac{5\pi}{32} \rho_0 u_0 \lambda_0 \tag{67}$$

In cgs units, this reference viscosity becomes

$$\eta_0 = 1.462 \times 10^{-5} \frac{\sqrt{T}}{1 + 112/T} \frac{gm}{cm-sec} \tag{68}$$

where T is the absolute temperature in degrees Kelvin. The ratio of mean velocities is

$$\frac{u_i}{u_0} = \sqrt{\frac{M_0}{M_i}} \tag{69}$$

and the ratio of densities, where the total number of moles per unit volume is the same, is

$$\frac{\rho_i}{\rho_0} = \frac{M_i}{M_0} x_i \tag{70}$$

Under these same conditions, that is, the same mol concentration, the ratio of mean free paths is (ref. 1)

$$\frac{\lambda_0}{\lambda_i} = \sum_j x_j \frac{S_{ij}}{S_0} \left(\frac{1 + \frac{M_i}{M_j}}{2} \right)^{1/2} \tag{71}$$

Then the ratio of viscosity to the reference viscosity becomes

$$\frac{\eta}{\eta_0} = \sum_i \sqrt{\frac{M_i}{M_0}} x_i \frac{\lambda_i}{\lambda_0} \tag{72}$$

Before equation (71) can be applied to the case where ionization occurs, it must be observed that the atom-electron collisions are not to be counted in evaluating the atom mean free paths, nor are the ion-electron collisions to be counted in evaluating the ion mean free paths. This is because the mean free path which occurs in the derivation of viscosity is the mean distance between those collisions which cause a relatively large change in momentum of the particle being considered, either in direction or in magnitude. Normally all collisions qualify in this sense, but when a heavy particle like an atom or an ion collides with an extremely light particle like an electron, there is practically no change in the momentum of the heavy particle, and this momentum is carried intact to the first collision with another heavy particle.

The coefficients of viscosity for high-temperature air have been calculated from equations (68), (71), and (72) and the results are presented in table VI(a) and in figure 8.

THERMAL CONDUCTIVITY

A theory for the thermal conductivity of a chemically reacting gas was perhaps first outlined by Nernst (ref. 27). The form in which this theory has been developed by Hirschfelder (ref. 17) will be used here to calculate the conductivity of high-temperature air. In this theory the energy transfer through the gas is treated in two independent parts. The first part is the energy transferred by molecular collisions, and this mode of energy transfer is the one responsible for the ordinary thermal conductivity of nonreacting gases. The second part is the energy transferred by diffusion of the molecular species and the reactions which occur as the gas tends to maintain itself in chemical equilibrium at each point. We shall first turn attention to the calculation of energy transferred by molecular collisions. If one invokes Eucken's assumption (ref. 28), that the internal energy is distributed among the gas particles independently of their velocity distribution, then the simple linearized expression for the coefficient of thermal conductivity in a mixed gas becomes

$$k_n = \frac{5\pi}{32} \sum_i \rho_i u_i \frac{\lambda_i}{M_i} \left(\frac{5}{2} C_i + C_{int} \right)_i \tag{73}$$

where C_i is the specific heat per mol due to translational energy and C_{int} the specific heat due to the internal energy. It is noted that

$$C_i = (C_i + C_{int})_i \tag{74}$$

and

$$C_t = \frac{3}{2} R \qquad (75)$$

whence equation (73) becomes

$$k_n = \frac{5\pi}{32} \sum_i \rho_i u_i \lambda_i \left(\frac{C_t}{M_i} + \frac{9}{4} \frac{R}{M_i} \right) \qquad (76)$$

It will be convenient to define a reference coefficient of thermal conductivity just as was done for the viscosity, that is, the value air would have if it did not vibrate, dissociate, or ionize. This coefficient will be

$$k_0 = \frac{19}{4} \frac{R}{M_0} \eta_0 \qquad (77)$$

or in cgs units

$$k_0 = 1.364 \, \eta_0 \, \frac{\text{joule}}{\text{cm-sec } °K} \qquad (78)$$

where η_0 is in gm/cm-sec. (eq. (68))

The ratio of the thermal conductivity coefficient to the reference coefficient becomes

$$\frac{k_n}{k_0} = \sum_i \left(\sqrt{\frac{M_i}{M_0}} \, x_i \frac{\lambda_i}{\lambda_0} \right) \frac{M_0}{M_i} \left[\frac{4}{19} \left(\frac{C}{R} \right)_i + \frac{9}{19} \right] \qquad (79)$$

The first factors in each term are just the terms already calculated for the viscosity. It may be noted that the same collisions are to be neglected in this case as for viscosity. This is because these electron-atom and electron-ion collisions have only a small effect on the magnitude and direction for the energy flux vector of the heavy particles. Actually, all of the electron collisions could have been neglected for the purpose of computing viscosity, for the electrons carry a negligible fraction of the total momentum transferred, due to their small mass. However, because of its high velocity, an electron transports a large share of the kinetic energy, in fact much more than a heavier gas particle. Therefore the heat conduction terms for the electrons are needed in equation (79).

The second mode of energy transfer, which takes place whenever the gas undergoes a chemical reaction, is due to the diffusion of the chemical species. These particles then react with one another, giving off or absorbing the heat of reaction and causing heat transfer which may be considerably larger than the ordinary heat transfer due to molecular collisions. Hirschfelder (ref. 17) has formulated this problem, but in terms of the
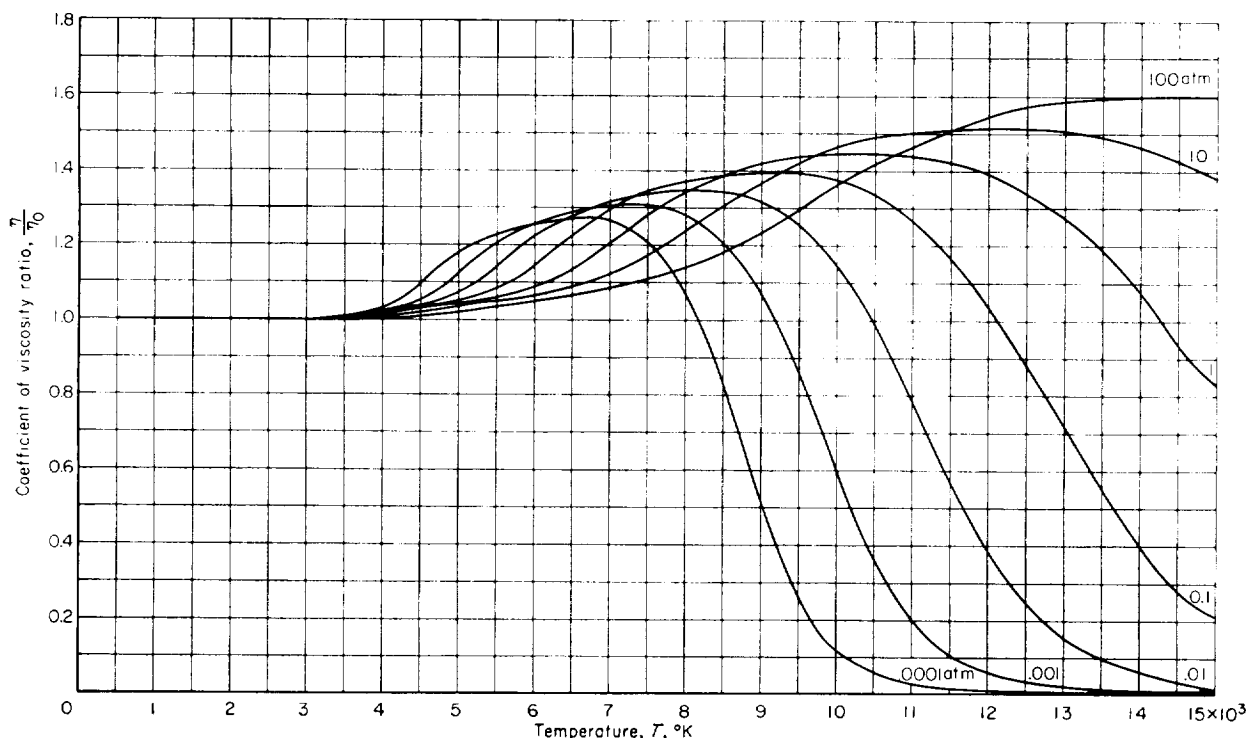
FIGURE 8.—Ratio of the coefficient of viscosity for air to the reference coefficient, η_0, as a function of temperature.

multicomponent diffusion coefficients which are difficult to estimate. However, Butler and Brokaw (ref. 29) have shown how Hirschfelder's results can be modified to make use of the binary diffusion coefficients instead, and their solutions are in convenient form for computational purposes. Moreover, Butler and Brokaw show that Hirschfelder's method of predicting thermal conductivity agrees well with the experimental results for a number of gases which dissociate or react chemically at ordinary temperatures. That part of the coefficient of thermal conductivity which is due to the chemical reaction will be designated k_r, and according to the results of reference 29, this may be expressed as

$$k_r = \frac{R\left(T\frac{d\ln K_p}{dT}\right)^2}{\sum_i \sum_j \frac{a_i}{nD_{ij}x_i}(a_i x_j - a_j x_i)} \tag{80}$$

where D_{ij} is the binary diffusion coefficient for molecules of type i into molecules of type j, and the a_i are the stoichiometric coefficients of components A_i in the chemical reaction written in the form

$$\sum_i a_i A_i = 0 \tag{81}$$

According to reference 1, the binary diffusion coefficients are given by

$$\frac{1}{nD_{ij}} = \frac{8}{3}\sqrt{\frac{2}{\pi}}\left(\frac{M_i M_j}{M_i+M_j}\right)^{1/2}\frac{N_0}{(RT)^{1/2}}S'_{ij} \tag{82}$$

Now the reference coefficient of thermal conductivity (eq. (77)) can be put in the form

$$k_0 = \frac{95\sqrt{\pi}}{64}\frac{R}{N_0 S_0}\sqrt{\frac{RT}{M_0}} \tag{83}$$

Substituting equation (82) in equation (80) and dividing by equation (83), one obtains

$$\frac{k_r}{k_0} = \frac{\frac{12\sqrt{2}}{95}\left(T\frac{d\ln K_p}{dT}\right)^2}{\sum_i \sum_j \left[\frac{M_i M_j}{M_0(M_i+M_j)}\right]^{1/2}\frac{S'_{ij}}{S_0}\frac{a_i}{x_i}(a_i x_j - a_j x_i)} \tag{84}$$

In order to simplify the computations somewhat, the differences in mass between oxygen and nitrogen atoms will be neglected. Then, since some of the collision cross sections may also be given the

same value, the double summations in the denominator of equation (84) take on the relatively simple form as follows:

For the oxygen dissociation reaction

$$\sum_i \sum_j = \frac{S'(N_2-N)}{\sqrt{3}S_0}\left\{\frac{[x(O)+2x(O_2)]^2}{x(O)x(O_2)} + \frac{4x(N_2)}{x(O)}\right\} + \frac{x(N_2)}{\sqrt{2}x(O_2)} \tag{85}$$

for the nitrogen dissociation reaction

$$\sum_i \sum_j = \frac{S'(N_2-N)}{\sqrt{3}S_0}\left\{\frac{[x(N)+2x(N_2)]^2}{x(N)x(N_2)} + \frac{x(O)}{x(N_2)}\right\} + \frac{S'(N-N)}{S_0}\frac{2x(O)}{x(N)} \tag{86}$$

and finally for the ionization reactions

$$\sum_i \sum_j = \left[\frac{1}{2}\frac{S'(N-N)}{S_0} + \frac{1}{2.30}\frac{S'(N-e^-)}{S_0}\right]\frac{[x(N)+x(N^+)]^2}{x(N)x(N^+)} \tag{87}$$

The total coefficient of thermal conductivity is just the sum, $k_n + k_r$. The values of this sum were calculated for air and the results are presented in table VI(b) and graphed in figure 9. The coefficient of thermal conductivity has about the same functional features with respect to temperature and pressure that were observed for the specific heat (fig. 5). The Prandtl numbers were calculated from the relation

$$Pr = \frac{C_p\eta}{Mk} = \frac{4}{19}\left(\frac{ZC_p}{R}\right)\frac{\eta/\eta_0}{k/k_0} \tag{88}$$

and they are listed in table VI(c) and are graphed as functions of temperature in figure 10.

As noted, nitrogen and oxygen have approximately equal collision cross sections, so that up to the point where ionization begins, air can be treated essentially as a two-component mixture of atoms and molecules. Then it is possible to characterize the system with a single diffusion coefficient, and the differential equations of fluid flow may be analyzed while keeping the terms describing heat transfer by chemical reaction separate from the terms describing ordinary thermal conduction. This method is followed, for example, by Fay and Riddell (ref. 30) in their analysis for the heat transfer to the stagnation
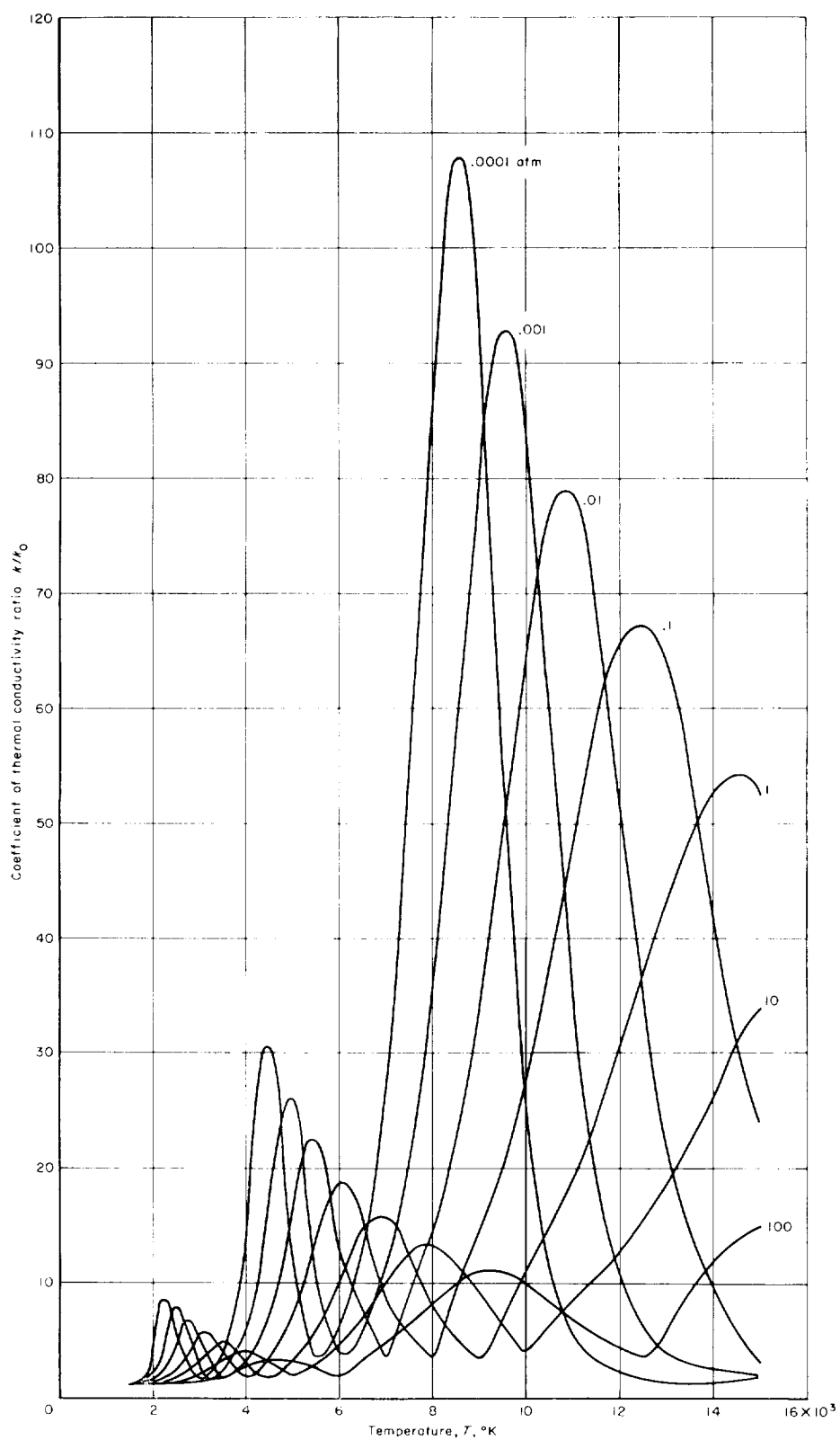
FIGURE 9.—Ratio of the coefficient of thermal conductivity of air to the reference coefficient, k_0, as a function of temperature.

region of blunt bodies in high-speed flight. In using such an analysis, it must be realized that the thermodynamic and transport coefficients are defined somewhat differently than in the present paper. The coefficients which appear in reference 30 will be called the "partial coefficients" and they will be designated here with a superscript prime. The partial specific heat, the partial coefficient of thermal conductivity, the partial Prandtl number, and the partial Lewis number, respectively, are related to the quantities defined in this paper as follows:

$$
\left.
\begin{aligned}
C_p' &= \sum x_i(C_i+R) \\
k' &= k_n \\
Pr' &= \frac{C_p'\eta}{Mk'} \\
Le' &= \frac{D\rho C_p'}{Mk'}
\end{aligned}
\right\}
\qquad (89)
$$

The partial Lewis number, Le', appears as a coefficient for the chemical reaction terms in

reference 30. The diffusion coefficient D is the ordinary binary diffusion coefficient D_{ij} given by equation (82), and from equation (83) a dimensionless group is

$$
\frac{ZD\rho R}{k_0 M_0}=0.309\left[\frac{S'(N_2-N)}{S_0}\right]^{-1}
\qquad (90)
$$

The Lewis number is conveniently calculated from the expression

$$
Le'=\frac{ZD\rho R}{k_0 M_0}\frac{ZC_p'/R}{Zk_n/k_0}
\qquad (91)
$$

Table VII lists the calculated values for the partial coefficients ZC_p'/R, k_n/k_0, Pr', and Le'. The numbers Pr' and Le' are also graphed as functions of temperature in figure 11. The partial coefficients are not given for temperatures where ionization occurs because the air is then at least a three-component system (neutral atoms, ionized atoms, and electrons) and the partial coefficients, as defined, do not apply to this case. It can be seen that Pr' is a relatively constant quantity with an average value about 0.72. The partial
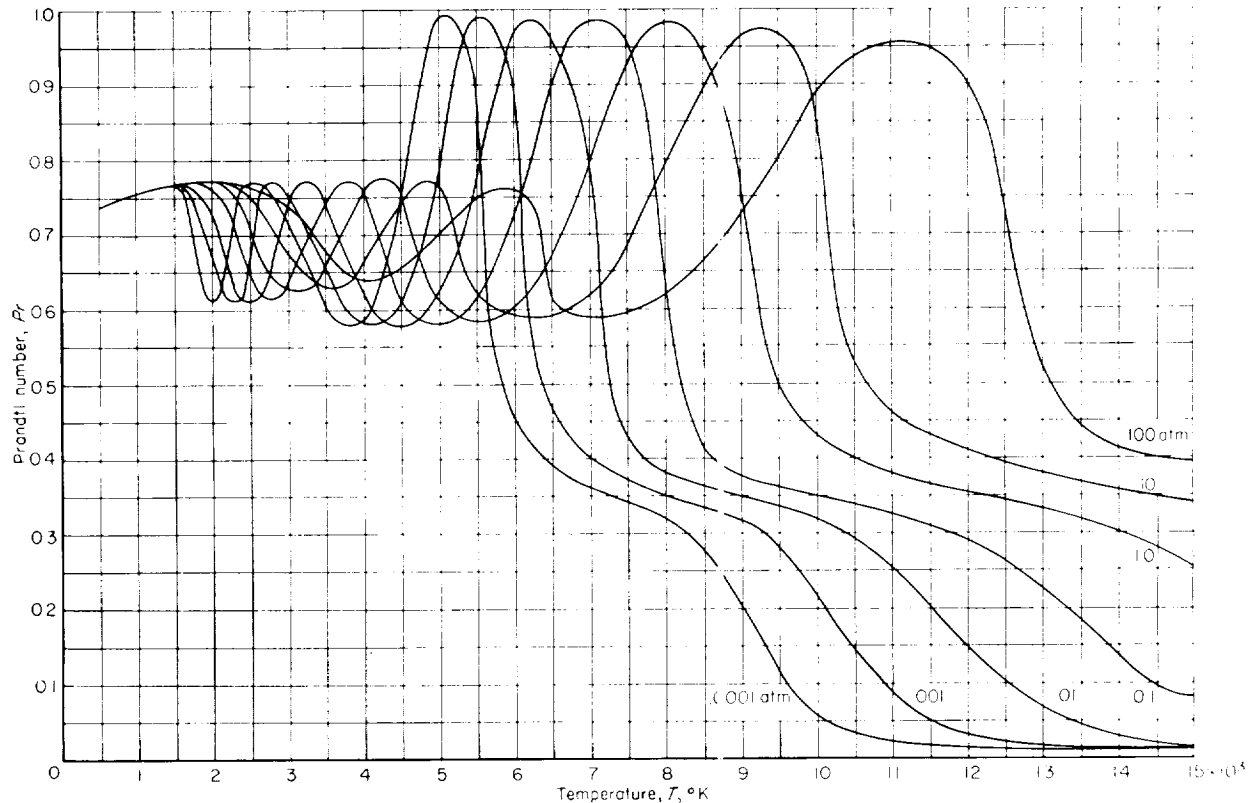
FIGURE 10.—Prandtl number for air as a function of temperature.

Lewis number, on the other hand, decreases as dissociation proceeds to completion. From the result that the partial Prandtl number is approximately constant, it follows that the factor $C_p'/\overline{M}k'$ decreases as η^{-1}. The factor $D\rho/\overline{M}$ increases at about the same rate (see eq. (82) or (90)), and
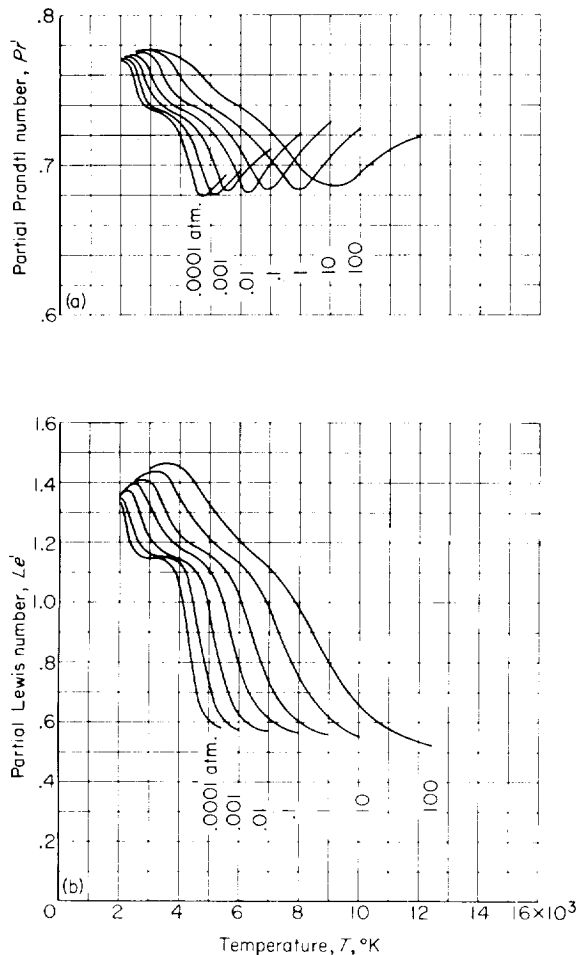
(a) Prandtl number.
(b) Lewis number.

FIGURE 11.—Partial coefficients for air as a function of temperature.

the product of these two factors is relatively constant; thus the partial Lewis number varies approximately as $\overline{M}$, that is, it decreases by a factor of about 2, from about 1.4 to 0.6.

A list of conversion factors is given in table VIII for convenience in converting the tabulated and graphed parameters into dimensioned units.

DISCUSSION OF RESULTS

It is convenient to summarize the physical processes which influence the transport properties of air in a discussion of the variations in Prandtl number with temperature. At low temperatures the air is like a pure diatomic gas with a specific heat about $7R/2$. From equation (88), the Prandtl number for such a gas is 14/19 or 0.74. As temperature increases, vibrational energy is excited in the molecules so that C_p approaches $9R/2$, while from equation (76) the thermal conductivity approaches $(23/4)(R\eta_0/M_0)$. For this limit the Prandtl number becomes 18/23 or 0.78.

At still higher temperatures the oxygen dissociates and both C_p and k go through the pronounced maxima shown in figures 5 and 9. To a
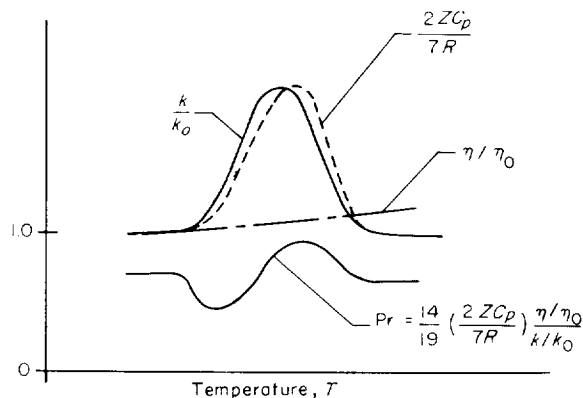
SKETCH (b).

first approximation, k is about proportional to C_p just as for a nonreacting gas. However, the maximum in k occurs at slightly lower temperatures than the maximum in specific heat as indicated in sketch (b). The viscosity coefficient is not greatly influenced by the oxygen dissociation, so from equation (88) it is apparent that the Prandtl number will follow an S shaped function as shown in sketch (b). This shape for the function can be seen in the curves of figure 10.

At the temperature where oxygen dissociation is essentially complete and nitrogen dissociation has not yet begun, the Prandtl number can be calculated for a two-component mixture from the known conditions that the mol fractions are $x_1=2/3$ and $x_2=1/3$ and that the molecular weights are related by $M_1=2M_2$ (where subscript 1 refers to nitrogen molecules and subscript 2 to oxygen

atoms). If the collision cross sections are all assumed equal, the mean free paths are related by

$$\frac{\lambda_2}{\lambda_1} = \frac{4+\sqrt{6}}{2+\sqrt{3}} \qquad (92)$$

Then the Prandtl number becomes

$$\left(\frac{C_p}{M}\right)\left(\frac{\eta}{k}\right) = \left(\frac{23}{5}\frac{R}{M_1}\right)\left[\frac{1+\frac{\sqrt{2}}{4}\frac{\lambda_2}{\lambda_1}}{\frac{R}{M_1}\left(\frac{23}{4}+\frac{15\sqrt{2}}{8}\frac{\lambda_2}{\lambda_1}\right)}\right] \simeq 0.72 \qquad (93)$$

which checks closely with the values for Prandtl number shown in figure 10 at the transitions from oxygen to nitrogen dissociation. ($Z=1.2$, fig. 1).

As nitrogen dissociation proceeds, the Prandtl number again follows an S shaped function of temperature for the same reasons which were outlined for oxygen dissociation (sketch (b)). The fully dissociated air is like a pure monatomic gas with C_p about $5R/2$ and k equal $(15/4)$ $(R\eta/M)$, so that the Prandtl number approaches $2/3$. The Prandtl numbers of figure 10 go through this value of $2/3$ at a temperature where the dissociation is essentially complete but the ionization is still negligible ($Z=2$, fig. 1).

Up to this point, the Prandtl numbers lie within the range from 0.6 to 1.0 in agreement with the conclusions of reference 31. However, when ionization begins, the Prandtl number may drop to somewhat lower values. For very small degrees of ionization it can be shown that the ratio of electron to atom mean free path is about $\sqrt{2}$ and that the coefficient of viscosity is influenced very little by the presence of the electrons. However, the thermal conductivity is greatly increased by the electrons because of their high thermal velocity, and the effect is illustrated in figure 9 by the abrupt change in slope for k where the ionization reaction first begins. If the fraction of ionization, ϵ, is so small that chemical reaction terms may be neglected

$$k \simeq \frac{15}{4}\frac{R}{M_1}\eta\left(1+\epsilon\sqrt{\frac{2M_1}{M_2}}\right) \qquad (94)$$

where M_1 is the atom mass and M_2 the electron mass. It can be seen that k increases rapidly with the onset of ionization because the factor $(2M_1/$

$M_2)^{1/2}$ is large, about 230. Accordingly the Prandtl number is found to vary as

$$\left(\frac{C_p}{M}\right)\left(\frac{\eta}{k}\right) = \left(\frac{5}{2}\frac{R}{M_1}\right)\left[\frac{15}{4}\frac{R}{M_1}\left(1+\epsilon\sqrt{\frac{2M_1}{M_2}}\right)\right]^{-1}$$
$$= \frac{2}{3(1+230\epsilon)} \qquad (95)$$

A very small value of ϵ is sufficient to reduce the Prandtl number considerably below $2/3$, the value for a pure monatomic gas. As the ionization proceeds, however, the chemical reaction terms which were omitted in equations (94) and (95) become predominant such that C_p becomes approximately proportional to k again, and the Prandtl number levels out at about 0.3 as shown in figure 10. At this point another factor gradually predominates, namely the decrease in coefficient of viscosity which occurs as the mean free paths become very short as a result of the long range coulomb forces acting on the charged gas particles. Consider, for example, the completely ionized gas. The mol faction of ions (subscript 1) equals the mol fraction of electrons (subscript 2) and the collision cross sections are about equal. Then the ratio of mean free paths is

$$\frac{\lambda_2}{\lambda_1} = \frac{\sqrt{2}}{1+\sqrt{2}} \qquad (96)$$

where as before, the electron collisions do not count toward the ion mean free paths. The ratio k/η is approximately given by

$$\frac{k}{\eta} \simeq \frac{15}{4}\frac{R}{M_1}\frac{\sqrt{2}}{1+\sqrt{2}}\sqrt{\frac{M_1}{M_2}} \qquad (97)$$

and with C_p/M about $5R/M_1$, the Prandtl number becomes very small

$$\frac{C_p}{M}\frac{\eta}{k} \simeq \frac{4}{3}\frac{1+\sqrt{2}}{\sqrt{2}}\sqrt{\frac{M_2}{M_1}}=0.014 \qquad (98)$$

as shown in figure 10. With the assumption that the Prandtl number retains its usual significance, boundary-layer regions in highly ionized flows, which are in complete thermal equilibrium, should be much better heat conductors than in the flow of neutral gas particles. This result is of theoretical interest even though fully ionized air flow will not be obtained at flight velocities below escape speed (fig. 7).

The Prandtl number functions of figure 10 are similar to those predicted by Kaeppeler and Krause (ref. 32). Their calculations were based on the low value for nitrogen dissociation energy and so the present estimates exhibit more distinctly separate maxima. This is because of the separation between the dissociation reactions which occurs when the higher value for nitrogen dissociation energy is used. Greifinger (ref. 33) has also made some estimates for the transport properties of air at high temperatures. He assumes constant collision cross sections in a manner such that, up to the point where ionization begins, his calculated Prandtl number is a constant lacking the structure shown in figure 10. However, reference 33 predicts an effect due to ionization which is the same order of magnitude as discussed here.

The neglect of the nitric oxide formation has only a small effect on the calculations of the thermodynamic properties and the viscosity of air. The thermal conductivity should be increased where the NO reaction occurs (see eq. (80)), but the heat of reaction is relatively small so that the correction involved is not large except at high densities. In addition, NO is ionized rather easily compared to N_2 and O_2 molecules and practically all of the electrons which appear in air at intermediate temperatures come from the NO ionization reaction. However, Gilmore finds that the mol fraction of such electrons is small, the order of 0.01 percent or less (ref. 9), and from equation (94) it can be judged that this amount is not yet sufficient to alter greatly the thermal conductivity. This degree of ionization is large, of course, in terms of its influence on electrical conductivity.

The theoretical transport coefficients can be used to estimate the heat flux to the stagnation region of a blunt body in high-energy air flow (see ref. 34). It is found that these predictions deviate less than 10 percent from the mean of the experimental results obtained by Rose and Riddell (ref. 35). This is a favorable result insofar as it fulfills a necessary condition for reliability of the calculated transport coefficients. It does not provide a sufficient check, since stagnation region heating is quite insensitive to real gas effects (ref. 34). However, some experimental determinations of the coefficient of thermal conductivity

integrated over temperature have been made up to 5000° K, through the range of oxygen dissociation. It is found (ref. 36) that the experiment and theory are in agreement within the accuracy expected of the elementary theoretical approach (the order of 30-percent uncertainty).

CONCLUDING REMARKS

In conclusion, the thermodynamic properties (including compressibility, energy, entropy, specific heat, and the speed of sound) and the transport properties (the coefficients of viscosity and thermal conductivity and the Prandtl number) have been estimated for air at high temperatures. These estimates were made from approximations in closed form, and it is found that these approximations give fairly accurately the thermodynamic properties over the range of pressures and temperatures for flight through the atmosphere. The transport properties were calculated by methods which have given reasonably good results for gases at ordinary temperatures and for gases which dissociate and react chemically. Moreover, the predicted coefficient of thermal conductivity integrated over temperature agrees reasonably well with the experimentally determined values up through conditions of oxygen dissociation. It is concluded then that the results presented in this paper are useful engineering approximations for the properties of high-temperature air. In particular, it is an advantage that the solutions appear in closed form because the functional relationships can be visualized. In addition the air properties can conveniently be computed in very small intervals to give tables that can be used in obtaining solutions to real air flow by the method of characteristics, or for numerical solutions of boundary-layer flow. It is expected that the calculated coefficients of viscosity and thermal conductivity will prove to be lower bounds to more precise calculations or measurements. The prediction that the Prandtl number for highly ionized flow in complete equilibrium will be small compared to unity implies that boundary-layer regions in such a gas will be very transparent to heat flux.

AMES RESEARCH CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MOFFETT FIELD, CALIF., *Nov. 18, 1957*

REFERENCES

1. Kennard, E. H.: Kinetic Theory of Gases. McGraw-Hill Book Co., 1938.
2. Eggers, A. J., Jr.: One-Dimensional Flows of an Imperfect Diatomic Gas. NACA Rep. 959, 1950. (Supersedes NACA TN 1861)
3. Herzberg, G.: Molecular Spectra and Molecular Structure I. Spectra of Diatomic Molecules. D. Van Nostrand Co., 1950.
4. Krieger, F. J., and White, W. B.: The Composition and Thermodynamic Properties of Air at Temperatures from 500 to 8000° K and Pressures from 0.00001 to 100 Atmospheres. Rand Rep. R–149, 1949.
5. Hirschfelder, J. O., and Curtiss, C. F.: Thermodynamic Properties of Air, I & II. Univ. of Wisconsin Dept. of Chemistry Reps. CM–472 and CM–518, 1948.
6. Gaydon, A. G.: Dissociation Energies and Spectra of Diatomic Molecules. Dover Pub., 1950.
7. Christian, R. H., Duff, R. E., and Yarger, F. L.: Equation of State of Gases by Shock Wave Measurements. II. The Dissociation Energy of Nitrogen. Jour. Chem. Phys., vol. 23, no. 11, Nov. 1955, pp. 2045–2049.
8. Kistiakowsky, G. B., Knight, Herbert T., and Malin, Murray E.: Gaseous Detonations. III. Dissociation Energies of Nitrogen and Carbon Monoxide. Jour. Chem. Phys., vol. 20, no. 5, May 1952, pp. 876–883.
9. Gilmore, F. R.: Equilibrium Composition and Thermodynamic Properties of Air to 24,000° K. Rand Rep. RM–1543, 1955.
10. Hilsenrath, J., and Beckett, Charles W.: Tables of Thermodynamic Properties of Argon-Free Air to 15,000° K. AEDC TN–56–12, 1956.
11. Logan, J. G., Jr., and Treanor, C. E.: Tables of Thermodynamic Properties of Air From 3,000° K to 10,000° K at Intervals of 100° K. Cornell Aero. Lab. Rep. BE–1007–A–3, Jan. 1957.
12. Hochstim, Adolf R.: Gas Properties Behind Shocks at Hypersonic Velocities. I. Normal Shocks in Air. Rep. No. ZPh(GP)–002, CONVAIR, Jan. 30, 1957.
13. Chapman, S., and Cowling, T. G.: The Mathematical Theory of Non-Uniform Gases. Cambridge U. Press, 1939.
14. Wang-Chang, C. S., and Uhlenbeck, G. E.: Transport Phenomena in Polyatomic Gases. Univ. of Michigan Rep. CM–681, 1951.
15. Hirschfelder, J. O., Curtiss, C. F., and Bird, R. B.: Molecular Theory of Gases and Liquids. J. Wiley and Sons, 1954.
16. Rose, P. H.: Physical Gas Dynamics Research at the AVCO Research Laboratory. AVCO Res. Rep. No. 9, 1957.
17. Hirschfelder, Joseph O.: Heat Transfer in Chemically Reacting Mixtures. I. Jour. Chem. Phys., vol. 26, no. 2, Feb. 1957, pp. 274–281.
18. Glasstone, Samuel: Theoretical Chemistry. D. Van Nostrand Co., 1944.
19. Mayer, J. E., and Mayer, M. G.: Statistical Mechanics. John Wiley and Sons, 1940.
20. Moore, C. E.: Atomic Energy Levels. Circular Nat. Bur. of Standards 467, 1948.
21. Feldman, Saul: Hypersonic Gas Dynamics Charts for Equilibrium Air. AVCO Res. Lab., Jan. 1957.
22. Hirschfelder, J. O., and Eliason, M. O.: The Estimation of the Transport Properties for Electronically Excited Atoms and Molecules. Univ. Wisconsin, Tech. Rep. WIS–AF–1, May 1956.
23. Hirschfelder, J. O., and Linnett, J. W.: The Energy of Interaction Between Two Hydrogen Atoms. Jour. Chem. Phys., vol. 18, no. 1, Jan. 1950, pp. 130–142.
24. Cohen, Robert S., Spitzer, Lyman, Jr., and Routly, Paul McR.: The Electrical Conductivity of an Ionized Gas. Phys. Rev., vol. 80, no. 2, Oct. 1950, pp. 230–238.
25. Joos, G.: Theoretical Physics. Hafner Co., 1932.
26. Gray, D. E., ed.: American Institute of Physics Handbook. McGraw-Hill Book Co., 1957.
27. Nernst, W.: Chemisches Gleichgewicht und Temperaturgefälle. Ludwig Boltzmann Festschrift, verlag von J. A. Barth, Leipzig, 1904. pp. 904–915.
28. Eucken, Arnold: On the Heat Conductivity, the Specific Heat, and the Internal Friction of Gases. Physikalische Zeitschift, vol. 14, no. 8, Apr. 1913, pp. 324–332.
29. Butler, James N., and Brokaw, Richard S.: Thermal Conductivity of Gas Mixtures in Chemical Equilibrium. Jour. Chem. Phys., vol. 26, no. 6, June 1957, pp. 1636–1643.
30. Fay, J. A., and Riddell, F. R.: Theory of Stagnation Point Heat Transfer in Dissociated Air. AVCO Res. Rep. No. 1, 1957.
31. Hansen, C. F.: Note on the Prandtl Number for Dissociated Air. Jour. Aero. Sci., vol. 20, no. 11, Nov. 1953, pp. 789–790.
32. Kneppeler, H. J., and Krause, H. G. L.: Thermodynamische Zustandsgrössen und Stoffwerte für luft bei Dissoziations-gleichgewicht. Mitteilungen aus dem Forschungsinstitut für Physik der Strahlantriebe, E. V., Stuttgart, 1954.
33. Greifinger, P. S.: Transport Coefficients of Dissociating and Slightly Ionizing Air. Rand Rep. RM–1794, 1957.
34. Eggers, Alfred J., Hansen, C. Frederick, and Cunningham, Bernard E.: Stagnation Point Heat Transfer to Blunt Shapes in Hypersonic Flight Including Effects of Yaw. NACA TN 4229, 1958. (Formerly NACA RM A55E02)
35. Rose, Peter H., and Riddell, F. R.: An Investigation of Stagnation Point Heat Transfer in Dissociated Air. AVCO Res. Rep. No. 7, 1957.
36. Hansen, C. Frederick, Early, Richard A., Alzofon, Frederick E., and Witteborn, Fred C.: Theoretical and Experimental Investigation of Heat Conduction in Air, Including Effects of Oxygen Dissociation. NASA Rep. 27, 1959.

TABLE I.—PARTITION FUNCTION CONSTANTS FOR THE MAJOR COMPONENTS OF AIR

Particle	Molecular weight, M, gm/mol	Rotational constant, $\frac{\alpha h^2}{8\pi^2 Ik}$, °K	Vibrational constant, $\frac{h\nu}{k}$, °K	Dissociation energy, $\frac{D}{k}$, °K	Electronic degeneracy, g_n	Electronic energy, $\frac{\epsilon_n}{k}$, °K	Ionization energy, $\frac{I}{k}$, °K
N₂	28	5.78	3390	113,200	1	0	
O₂	32	4.16	2270	59,000	3 2 1	0 11,390 18,990	
O	16				5 3 1 5 1	0 228 326 22,800 48,600	158,000
N	14				4 10 6	0 27,700 41,500	168,800
O⁺	16				4 10 6	0 38,600 58,200	
N⁺	14				1 3 5 5 1 5	0 70.6 188.9 22,000 47,000 67,900	
e⁻	$\frac{1}{1820}$				2	0	

TABLE II.—THERMODYNAMIC PROPERTIES FOR THE MAJOR COMPONENTS OF AIR

T, °K	(a) Logarithm of the partition function for a standard state of one atmosphere, ln Q_p — N₂	O₂	N	O	N⁺,O⁺	e⁻	(b) Dimensionless enthalpy, $\frac{H-E_0}{RT}$ — N₂	O₂	N	O	N⁺, O⁺	e⁻	(c) Dimensionless specific heat, $\frac{1}{R}\frac{dH}{dT}$ — N₂	O₂	N	O	N⁺, O⁺	e⁻
500	21.33	22.97	17.22	18.04	17.71	1.30	3.51	3.55	2.50	2.66	2.69	2.50	3.55	3.72	2.50	2.56	2.52	2.50
1000	23.79	25.49	18.96	19.86	19.54	3.03	3.62	3.76	2.50	2.60	2.60	2.50	3.91	4.16	2.50	2.52	2.50	2.50
1500	25.29	27.05	19.97	20.91	20.59	4.04	3.76	3.93	2.50	2.57	2.57	2.50	4.16	4.35	2.50	2.51	2.50	2.50
2000	26.39	28.20	20.69	21.65	21.33	4.76	3.88	4.05	2.50	2.55	2.55	2.50	4.29	4.47	2.50	2.50	2.50	2.50
2500	27.26	29.12	21.25	22.21	21.89	5.32	3.97	4.15	2.50	2.54	2.54	2.50	4.36	4.59	2.50	2.51	2.51	2.50
3000	27.99	29.88	21.70	22.68	22.36	5.78	4.04	4.23	2.50	2.54	2.54	2.50	4.40	4.69	2.50	2.52	2.52	2.50
3500	28.62	30.54	22.09	23.07	22.75	6.16	4.09	4.30	2.51	2.54	2.53	2.50	4.43	4.77	2.56	2.54	2.53	2.50
4000	29.17	31.12	22.42	23.41	23.09	6.50	4.14	4.36	2.52	2.54	2.54	2.50	4.44	4.82	2.62	2.56	2.56	2.50
4500	29.66	31.63	22.72	23.71	23.39	6.79	4.17	4.42	2.53	2.54	2.54	2.50	4.45	4.86	2.71	2.59	2.59	2.50
5000	30.10	32.10	22.99	23.97	23.65	7.05	4.20	4.46	2.56	2.55	2.55	2.50	4.46	4.88	2.82	2.62	2.62	2.50
5500	30.50	32.53	23.23	24.22	23.90	7.29	4.22	4.50	2.59	2.56	2.55	2.50	4.47	4.89	2.94	2.65	2.65	2.50
6000	30.87	32.92	23.46	24.44	24.12	7.51	4.24	4.53	2.62	2.57	2.56	2.50	4.47	4.89	3.07	2.68	2.69	2.50
6500	31.21	33.28	23.67	24.65	24.32	7.71	4.26	4.56	2.66	2.58	2.56	2.50	4.48	4.89	3.19	2.70	2.72	2.50
7000	31.53	33.62	23.87	24.84	24.52	7.89	4.28	4.58	2.70	2.59	2.59	2.50	4.48	4.88	3.30	2.72	2.75	2.50
7500	31.82	33.94	24.06	25.02	24.69	8.07	4.29	4.60	2.75	2.60	2.60	2.50	4.48	4.86	3.40	2.74	2.79	2.50
8000	32.10	34.24	24.24	25.18	24.86	8.23	4.30	4.62	2.79	2.60	2.61	2.50	4.49	4.85	3.49	2.75	2.82	2.50
8500	32.36	34.52	24.41	25.34	25.02	8.38	4.31	4.63	2.83	2.61	2.62	2.50	4.49	4.83	3.56	2.77	2.84	2.50
9000	32.61	34.78	24.57	25.49	25.17	8.52	4.32	4.64	2.87	2.62	2.64	2.50	4.49	4.82	3.61	2.77	2.87	2.50
9500	32.84	35.03	24.73	25.63	25.32	8.66	4.33	4.65	2.91	2.63	2.65	2.50	4.49	4.80	3.64	2.78	2.89	2.50
10000	33.06	35.27	24.88	25.77	25.45	8.79	4.34	4.66	2.95	2.64	2.66	2.50	4.49	4.79	3.66	2.78	2.92	2.50
10500	33.27	35.50	25.02	25.90	25.58	8.91	4.35	4.66	2.98	2.65	2.68	2.50	4.49	4.77	3.67	2.78	2.94	2.50
11000	33.48	35.72	25.16	26.02	25.71	9.02	4.35	4.67	3.02	2.65	2.69	2.50	4.49	4.76	3.66	2.78	2.95	2.50
11500	33.67	35.92	25.30	26.14	25.83	9.14	4.36	4.67	3.04	2.66	2.70	2.50	4.49	4.74	3.65	2.78	2.97	2.50
12000	33.86	36.12	25.43	26.25	25.94	9.24	4.37	4.67	3.07	2.66	2.71	2.50	4.49	4.73	3.63	2.78	2.98	2.50
12500	34.03	36.31	25.55	26.36	26.05	9.34	4.37	4.68	3.09	2.67	2.72	2.50	4.49	4.72	3.61	2.78	2.99	2.50
13000	34.21	36.50	25.67	26.47	26.16	9.44	4.38	4.68	3.11	2.67	2.73	2.50	4.49	4.71	3.58	2.78	3.00	2.50
13500	34.37	36.67	25.79	26.57	26.26	9.54	4.38	4.68	3.13	2.68	2.74	2.50	4.49	4.70	3.55	2.77	3.00	2.50
14000	34.53	36.84	25.91	26.66	26.36	9.63	4.38	4.68	3.14	2.68	2.75	2.50	4.50	4.69	3.52	2.77	3.01	2.50
14560	34.68	37.01	26.02	26.76	26.46	9.71	4.39	4.68	3.15	2.68	2.76	2.50	4.50	4.68	3.48	2.76	3.01	2.50
15000	34.83	37.17	26.12	26.85	26.55	9.80	4.39	4.68	3.16	2.68	2.77	2.50	4.50	4.67	3.45	2.76	3.01	2.50

TABLE III.—LOGARITHMS OF THE CHEMICAL EQUILIBRIUM CONSTANTS AND THEIR DERIVATIVES FOR THE MAJOR REACTIONS IN AIR

| T, °K | (a) $\ln K_p$ | | | (b) $T \dfrac{d \ln K_p}{dT}$ | | |
| | Reaction | | | Reaction | | |
	$O_2 \rightarrow 2O$	$N_2 \rightarrow 2N$	$N \rightarrow N^+ + e^-$ $O \rightarrow O^+ + e^-$	$O_2 \rightarrow 2O$	$N_2 \rightarrow 2N$	$N \rightarrow N^+ + e^-$ $O \rightarrow O^+ + e^-$
500	−104.89	−213.29	−331.58	119.78	227.89	335.86
1000	−44.77	−99.08	−163.16	60.43	114.58	169.18
1500	−24.57	−60.81	−106.59	40.54	76.70	113.62
2000	−14.41	−41.61	−78.09	30.55	57.72	85.84
2500	−8.29	−30.05	−60.87	24.54	46.31	69.17
3000	−4.19	−22.32	−49.30	20.51	38.70	58.06
3500	−1.26	−16.79	−40.98	17.63	33.26	50.12
4000	.95	−12.62	−34.69	15.46	29.20	44.16
4500	2.67	−9.37	−29.77	13.78	26.05	39.53
5000	4.05	−6.76	−25.80	12.44	23.55	35.81
5500	5.18	−4.61	−22.53	11.34	21.53	32.77
6000	6.13	−2.81	−19.80	10.43	19.87	30.22
6500	6.93	−1.28	−17.46	9.67	18.47	28.06
7000	7.62	.05	−15.45	9.02	17.30	26.21
7500	8.23	1.20	−13.70	8.45	16.29	24.60
8000	8.75	2.23	−12.16	7.97	15.43	23.18
8500	9.22	3.14	−10.79	7.54	14.67	21.94
9000	9.64	3.96	−9.57	7.16	14.00	20.82
9500	10.02	4.70	−8.47	6.82	13.41	19.83
10000	10.36	5.37	−7.48	6.52	12.88	18.93
10500	10.68	5.99	−6.57	6.25	12.40	18.13
11000	10.96	6.56	−5.75	6.00	11.97	17.39
11500	11.22	7.08	−4.99	5.77	11.57	16.72
12000	11.46	7.56	−4.29	5.57	11.21	16.11
12500	11.69	8.01	−3.65	5.38	10.87	15.54
13000	11.89	8.44	−3.05	5.20	10.55	15.03
13500	12.09	8.83	−2.49	5.04	10.26	14.55
14000	12.27	9.20	−1.97	4.89	9.99	14.10
14500	12.44	9.54	−1.48	4.75	9.73	13.69
15000	12.60	9.87	−1.02	4.62	9.48	13.31

TABLE IV.—THERMODYNAMIC PROPERTIES FOR AIR—Concluded

T, °K	(e) Ratio of specific heats, γ							(f) Dimensionless speed of sound parameter, $\dfrac{a^2\rho}{p}$						
	Pressure, atmospheres							Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001	100	10	1.0	0.1	0.01	0.001	0.0001
500	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387	1.387
1000	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337	1.337
1500	1.312	1.312	1.312	1.312	1.312	1.310	1.306	1.312	1.312	1.312	1.312	1.312	1.310	1.306
2000	1.300	1.299	1.296	1.286	1.260	1.209	1.153	1.300	1.299	1.296	1.285	1.259	1.206	1.144
2500	1.288	1.277	1.249	1.202	1.161	1.152	1.157	1.288	1.276	1.247	1.196	1.144	1.119	1.132
3000	1.266	1.235	1.195	1.178	1.181	1.239	1.304	1.265	1.229	1.181	1.147	1.150	1.232	1.302
3500	1.241	1.211	1.202	1.212	1.270	1.252	1.176	1.234	1.192	1.166	1.187	1.265	1.250	1.171
4000	1.230	1.223	1.230	1.260	1.213	1.150	1.133	1.212	1.187	1.204	1.254	1.208	1.137	1.097
4500	1.240	1.243	1.251	1.204	1.154	1.155	1.190	1.208	1.210	1.241	1.196	1.133	1.101	1.092
5000	1.256	1.252	1.212	1.166	1.172	1.203	1.168	1.217	1.233	1.202	1.143	1.111	1.103	1.124
5500	1.262	1.231	1.183	1.182	1.214	1.183	1.257	1.229	1.217	1.161	1.124	1.113	1.133	1.249
6000	1.253	1.206	1.190	1.221	1.202	1.237	1.266	1.230	1.186	1.141	1.124	1.135	1.225	1.263
6500	1.235	1.201	1.220	1.228	1.217	1.265	1.188	1.214	1.165	1.137	1.136	1.193	1.260	1.183
7000	1.223	1.217	1.246	1.216	1.258	1.210	1.155	1.195	1.155	1.142	1.167	1.249	1.205	1.140
7500	1.223	1.243	1.244	1.237	1.237	1.173	1.164	1.182	1.154	1.156	1.216	1.231	1.162	1.128
8000	1.235	1.264	1.235	1.252	1.201	1.168	1.201	1.175	1.159	1.181	1.242	1.193	1.143	1.130
8500	1.255	1.267	1.243	1.235	1.183	1.188	1.242	1.174	1.169	1.214	1.228	1.168	1.140	1.136
9000	1.275	1.260	1.252	1.213	1.185	1.224	1.244	1.176	1.185	1.237	1.203	1.157	1.144	1.145
9500	1.288	1.255	1.248	1.201	1.203	1.256	1.216	1.181	1.206	1.237	1.185	1.155	1.151	1.157
10000	1.291	1.259	1.236	1.201	1.232	1.263	1.211	1.190	1.228	1.225	1.176	1.159	1.159	1.185
10500	1.287	1.262	1.226	1.213	1.263	1.244	1.256	1.201	1.242	1.212	1.173	1.165	1.169	1.244
11000	1.282	1.261	1.222	1.233	1.281	1.230	1.339	1.216	1.246	1.202	1.175	1.173	1.187	1.334
11500	1.280	1.256	1.226	1.258	1.280	1.243	1.427	1.232	1.242	1.198	1.180	1.181	1.221	1.424
12000	1.282	1.252	1.236	1.283	1.267	1.288	1.491	1.247	1.236	1.197	1.187	1.192	1.276	1.489
12500	1.284	1.250	1.251	1.301	1.257	1.352	1.528	1.257	1.231	1.198	1.194	1.207	1.346	1.528
13000	1.285	1.251	1.271	1.307	1.263	1.419	1.548	1.263	1.227	1.202	1.202	1.232	1.415	1.548
13500	1.284	1.257	1.291	1.303	1.288	1.472	1.558	1.265	1.226	1.208	1.210	1.269	1.470	1.558
14000	1.284	1.266	1.311	1.295	1.329	1.509	1.563	1.265	1.226	1.214	1.221	1.317	1.508	1.563
14500	1.284	1.278	1.326	1.290	1.377	1.532	1.565	1.264	1.228	1.221	1.235	1.370	1.532	1.565
15000	1.286	1.293	1.336	1.263	1.425	1.547	1.567	1.263	1.232	1.228	1.254	1.420	1.546	1.567

TABLE V.—COLLISION CROSS SECTIONS

T, °K	S_0, in 10^{-16} cm²	$\dfrac{S(N_2-N)}{S_0}$	$\dfrac{S(N-N)}{S_0}$	$\dfrac{S(N-e)}{S_0}$	$\dfrac{S(e-e)}{S_0 \ln \Lambda}$	$\dfrac{S'(N_2-N)}{S_0}$	$\dfrac{S'(N-N)}{S_0}$
500	38.4	0.946	0.894			0.877	0.761
1,000	34.9	.920	.838			.843	.703
1,500	33.7	.889	.785			.817	.652
2,000	33.2	.886	.742			.794	.611
2,500	32.8	.846	.705			.775	.578
3,000	32.6	.830	.676			.759	.551
3,500	32.4	.815	.650			.745	.527
4,000	32.3	.803	.628			.733	.507
4,500	32.2	.792	.608			.722	.489
5,000	32.1	.782	.591			.712	.473
5,500	32.0	.773	.575	0.397	89.9	.703	.458
6,000	32.0	.764	.561	.380	75.6	.695	.445
6,500	31.9	.757	.548	.366	64.5	.688	.433
7,000	31.9	.750	.536	.353	55.7	.681	.422
7,500	31.9	.743	.524	.342	48.6	.674	.412
8,000	31.8	.737	.514	.331	42.8	.668	.402
8,500	31.8	.731	.504	.321	37.9	.662	.393
9,000	31.8	.725	.495	.313	33.8	.657	.385
9,500	31.8	.720	.486	.304	30.4	.652	.377
10,000	31.8	.715	.478	.297	27.4	.647	.370
10,500	31.7	.710	.470	.290	24.9	.642	.363
11,000	31.7	.706	.463	.283	22.7	.637	.356
11,500	31.7	.701	.456	.281	20.8	.633	.350
12,000	31.7	.697	.448	.270	19.09	.629	.342
12,500	31.7	.693	.443	.266	17.60	.625	.338
13,000	31.7	.689	.437	.261	16.27	.621	.332
13,500	31.7		.431	.256	15.10		.327
14,000	31.7		.426	.256	14.04		.322
14,500	31.6		.420	.252	13.09		.316
15,000	31.6		.415	.247	12.24		.312

$S(N_2-N)=S(O_2-O)=S(N_2-O)$
$S(N-N)=S(O-O)=S(N-O)=S(N-N^+)=S(O-O^+)$
$S(N-e)=S(O-e)$
$S'(N_2-N)=S'(O_2-O)=S'(N_2-O)$
$S'(N-N)=S'(O-O)=S'(N-O)=S'(N-N^+)=S'(O-O^+)$

TABLE IV.—THERMODYNAMIC PROPERTIES FOR AIR

(a) Compressibility, Z

T, °K	Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1500	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2000	1.000	1.000	1.000	1.001	1.002	1.005	1.016
2500	1.000	1.001	1.004	1.011	1.033	1.088	1.163
3000	1.003	1.009	1.026	1.072	1.149	1.192	1.200
3500	1.012	1.035	1.092	1.167	1.197	1.203	1.211
4000	1.033	1.089	1.165	1.198	1.208	1.228	1.287
4500	1.071	1.149	1.196	1.213	1.245	1.337	1.577
5000	1.118	1.186	1.214	1.252	1.359	1.622	1.910
5500	1.159	1.208	1.248	1.348	1.599	1.898	1.990
6000	1.189	1.235	1.316	1.529	1.849	1.983	2.008
6500	1.214	1.279	1.437	1.752	1.961	2.006	2.032
7000	1.243	1.351	1.607	1.904	1.997	2.027	2.088
7500	1.284	1.457	1.778	1.971	2.017	2.067	2.210
8000	1.341	1.590	1.896	2.001	2.044	2.144	2.446
8500	1.418	1.727	1.959	2.023	2.090	2.284	2.826
9000	1.512	1.838	1.993	2.050	2.166	2.510	3.282
9500	1.616	1.914	2.018	2.090	2.286	2.832	3.645
10000	1.718	1.962	2.042	2.149	2.462	3.202	3.843
10500	1.807	1.993	2.071	2.234	2.700	3.526	3.932
11000	1.876	2.018	2.111	2.351	2.983	3.745	3.969
11500	1.927	2.042	2.163	2.505	3.272	3.867	3.985
12000	1.965	2.067	2.232	2.694	3.520	3.931	3.993
12500	1.993	2.098	2.318	2.910	3.700	3.963	3.996
13000	2.017	2.135	2.426	3.135	3.818	3.979	3.998
13500	2.039	2.180	2.553	3.347	3.889	3.988	3.999
14000	2.062	2.233	2.700	3.527	3.932	3.993	3.999
14500	2.086	2.297	2.861	3.667	3.957	3.996	4.000
15000	2.113	2.372	3.028	3.769	3.973	3.997	4.000

(b) Dimensionless enthalpy, $\frac{ZH}{RT}$

T, °K	Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001
500	3.52	3.52	3.52	3.52	3.52	3.52	3.52
1000	3.65	3.65	3.65	3.65	3.65	3.65	3.65
1500	3.80	3.80	3.80	3.80	3.80	3.80	3.80
2000	3.92	3.92	3.92	3.93	3.97	4.07	4.41
2500	4.01	4.03	4.09	4.27	4.81	6.16	8.02
3000	4.13	4.25	4.61	5.55	7.13	8.02	8.19
3500	4.34	4.75	5.75	7.08	7.62	7.77	8.03
4000	4.70	5.56	6.74	7.28	7.53	8.09	9.82
4500	5.20	6.29	6.98	7.33	8.14	10.55	16.80
5000	5.73	6.62	7.10	7.96	10.48	16.68	23.46
5500	6.13	6.80	7.58	9.73	15.14	21.58	23.58
6000	6.38	7.11	8.70	12.93	19.30	21.97	22.54
6500	6.62	7.72	10.64	16.46	20.35	21.24	21.93
7000	6.95	8.76	13.20	18.34	20.01	20.69	22.29
7500	7.44	10.24	15.48	18.66	19.54	20.72	24.26
8000	8.16	11.99	16.73	18.43	19.34	21.65	28.65
8500	9.10	13.63	17.09	18.17	19.60	23.85	35.75
9000	10.20	14.79	17.04	18.09	20.49	27.66	43.74
9500	11.36	15.40	16.91	18.29	22.17	33.00	49.15
10000	12.42	15.61	16.84	18.85	24.78	38.79	50.96
10500	13.23	15.64	16.90	19.84	28.28	43.28	50.64
11000	13.77	15.60	17.13	21.31	32.31	45.57	49.48
11500	14.08	15.58	17.57	23.28	36.13	46.09	48.07
12000	14.22	15.62	18.24	25.69	39.01	45.64	46.64
12500	14.28	15.74	19.16	28.36	40.66	44.74	45.26
13000	14.30	15.96	20.32	30.99	41.26	43.69	43.97
13500	14.31	16.28	21.72	33.27	41.17	42.61	42.76
14000	14.34	16.71	23.29	34.97	40.69	41.55	41.64
14500	14.40	17.26	24.98	36.02	40.01	40.53	40.58
15000	14.49	17.92	26.66	36.53	39.24	39.57	39.60

(c) Dimensionless entropy, $\frac{ZS}{R}$

T, °K	100	10	1.0	0.1	0.01	0.001	0.0001
500	21.1	23.4	25.7	28.0	30.3	32.6	34.9
1000	23.7	26.0	28.3	30.6	32.9	35.2	37.5
1500	25.3	27.6	29.9	32.2	34.5	36.8	39.2
2000	26.6	28.9	31.2	33.5	35.8	38.2	40.9
2500	27.5	29.9	32.2	34.7	37.6	41.4	45.9
3000	28.4	30.8	33.5	36.9	41.0	44.6	47.5
3500	29.3	32.0	35.5	39.4	42.7	45.6	48.6
4000	30.2	33.5	37.3	40.6	43.6	46.9	51.6
4500	31.3	34.9	38.3	41.5	45.1	50.5	60.0
5000	32.4	36.0	39.2	42.9	48.4	58.0	68.9
5500	33.4	36.8	40.4	45.5	54.3	64.8	71.3
6000	34.2	37.7	42.2	49.7	59.9	67.1	72.2
6500	34.9	38.9	44.9	54.4	62.6	68.1	73.4
7000	35.8	40.5	48.3	57.6	63.8	69.1	75.4
7500	36.8	42.7	51.6	59.2	64.6	70.5	79.0
8000	38.0	45.1	53.9	60.1	65.7	72.8	85.0
8500	39.4	47.6	55.3	61.0	67.1	76.4	94.1
9000	41.1	49.5	56.2	61.9	69.2	81.6	104.3
9500	42.8	51.0	57.0	63.1	72.0	88.6	112.2
10000	44.5	52.0	57.8	64.6	75.8	96.2	116.6
10500	45.9	52.8	58.7	66.6	80.6	102.7	118.8
11000	47.1	53.4	59.7	69.0	86.0	107.1	119.9
11500	48.0	54.1	60.9	71.9	91.4	109.6	120.7
12000	48.8	54.8	62.4	75.1	95.8	111.1	121.3
12500	49.4	55.6	64.0	79.1	99.1	112.1	121.8
13000	50.0	56.4	66.0	82.9	101.3	112.8	122.3
13500	50.5	57.3	68.2	86.4	102.8	113.3	122.7
14000	51.1	58.4	70.5	89.4	103.8	113.8	123.1
14500	51.7	59.5	73.1	91.7	104.5	114.2	123.5
15000	52.2	60.8	75.6	93.4	105.1	114.6	123.9

(d) Dimensionless specific heat at constant pressure, $\frac{ZC_p}{R}$

T, °K	100	10	1.0	0.1	0.01	0.001	0.0001
500	3.59	3.59	3.59	3.59	3.59	3.59	3.59
1000	3.96	3.96	3.96	3.96	3.96	3.96	3.96
1500	4.20	4.20	4.20	4.20	4.21	4.23	4.29
2000	4.41	4.38	4.41	4.57	5.10	6.73	11.57
2500	4.51	4.73	5.46	7.63	13.58	24.00	20.03
3000	5.03	6.23	9.62	16.64	17.66	7.88	5.41
3500	6.27	9.50	14.90	12.31	6.51	6.64	10.69
4000	8.24	12.47	11.05	6.91	8.49	16.46	40.78
4500	10.12	11.06	7.63	9.61	19.68	49.06	101.49
5000	10.53	8.57	9.58	18.98	46.22	87.18	47.49
5500	9.56	9.06	15.85	37.24	72.70	44.58	12.82
6000	9.05	12.3	26.95	57.60	49.37	15.16	11.43
6500	10.05	18.3	41.13	54.37	20.60	11.76	19.13
7000	12.59	26.5	49.61	31.18	12.89	16.38	36.82
7500	16.48	35.1	42.69	17.32	13.99	27.02	70.02
8000	21.42	40.3	28.27	13.68	19.40	45.62	121.60
8500	26.70	38.1	18.58	14.91	28.85	74.32	173.36
9000	31.05	30.3	14.83	18.90	43.11	111.46	173.32
9500	32.90	22.5	14.65	25.26	62.69	143.94	114.86
10000	31.43	17.4	16.53	34.11	86.43	147.10	60.29
10500	27.46	15.1	19.86	45.53	109.34	114.76	31.99
11000	22.82	14.8	24.46	59.25	121.78	73.50	19.92
11500	18.96	15.7	30.27	74.10	115.48	44.18	14.93
12000	16.39	17.4	37.23	87.57	93.38	27.91	12.82
12500	15.01	19.8	45.17	96.01	67.36	19.67	11.89
13000	14.57	22.8	53.70	96.15	46.35	15.56	11.47
13500	14.80	26.4	62.12	87.59	32.30	13.49	11.26
14000	15.52	30.4	69.41	73.42	23.72	12.41	11.16
14500	16.62	31.7	74.36	58.06	18.66	11.83	11.10
15000	18.02	39.3	75.95	44.65	15.70	11.51	11.08

TABLE VI.—TRANSPORT PROPERTIES OF AIR

(a) Coefficient of viscosity

T, °K	Ratio η/η_0, Pressure, atmospheres							Reference coefficient, η_0, in 10^{-6}	
	100	10	1.0	0.1	0.01	0.001	0.0001	$\dfrac{\text{lb sec}}{\text{ft}^2}$	$\dfrac{\text{gm}}{\text{cm sec}}$
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.558	267
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.868	416
1500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.100	527
2000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.293	619
2500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.461	700
3000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.612	772
3500	1.000	1.001	1.003	1.006	1.010	1.010	1.011	1.751	838
4000	1.003	1.008	1.016	1.020	1.022	1.024	1.032	1.879	899
4500	1.010	1.022	1.029	1.033	1.038	1.055	1.096	1.999	957
5000	1.022	1.036	1.043	1.051	1.074	1.128	1.181	2.11	1011
5500	1.036	1.052	1.060	1.086	1.146	1.209	1.227	2.22	1062
6000	1.050	1.067	1.090	1.148	1.228	1.257	1.256	2.32	1112
6500	1.072	1.090	1.139	1.229	1.276	1.286	1.271	2.42	1159
7000	1.089	1.124	1.208	1.294	1.317	1.303	1.264	2.52	1204
7500	1.112	1.175	1.283	1.332	1.337	1.307	1.210	2.61	1247
8000	1.143	1.238	1.342	1.371	1.347	1.280	1.072	2.69	1289
8500	1.185	1.307	1.386	1.386	1.343	1.207	.826	2.78	1330
9000	1.238	1.368	1.425	1.396	1.314	1.068	.517	2.86	1370
9500	1.298	1.418	1.438	1.393	1.251	.853	.261	2.94	1408
10000	1.361	1.468	1.445	1.375	1.143	.595	.118	3.02	1446
10500	1.418	1.496	1.448	1.335	.983	.361	.055	3.10	1482
11000	1.467	1.501	1.442	1.267	.782	.200	.029	3.17	1518
11500	1.509	1.511	1.424	1.168	.571	.108	.018	3.24	1552
12000	1.549	1.520	1.394	1.040	.387	.063	.012	3.31	1586
12500	1.577	1.516	1.342	.881	.249	.036	.009	3.38	1620
13000	1.581	1.508	1.274	.711	.158	.024	.008	3.45	1652
13500	1.594	1.492	1.187	.547	.100	.018	.007	3.52	1684
14000	1.599	1.468	1.082	.408	.067	.015	.007	3.58	1716
14500	1.601	1.415	.940	.268	.042	.013	.008	3.65	1747
15000	1.604	1.387	.828	.212	.016	.012	.008	3.71	1777

(b) Coefficient of thermal conductivity

T, °K	Ratio k/k_0, Pressure, atmospheres							Reference coefficient, k_0, in 10^{-6}	
	100	10	1.0	0.1	0.01	0.001	0.0001	$\dfrac{\text{Btu}}{\text{ft sec }°R}$	$\dfrac{\text{watt}}{\text{cm }°K}$
500	1.021	1.021	1.021	1.021	1.021	1.021	1.021	5.84	364
1000	1.100	1.100	1.100	1.100	1.100	1.100	1.100	9.10	567
1500	1.150	1.150	1.150	1.150	1.150	1.150	1.150	11.53	719
2000	1.177	1.177	1.177	1.251	1.460	2.09	3.99	13.55	844
2500	1.256	1.317	1.619	2.50	4.63	7.67	5.50	15.31	954
3000	1.421	1.928	3.20	5.48	5.02	2.19	1.465	16.90	1053
3500	1.941	3.15	4.72	3.96	1.719	2.11	3.71	18.35	1143
4000	2.69	3.94	2.99	1.600	2.91	6.04	15.03	19.60	1227
4500	3.22	3.06	1.714	3.32	7.34	17.65	30.5	21.0	1305
5000	3.07	1.997	3.29	7.18	16.63	25.8	11.84	22.1	1379
5500	2.46	2.91	5.99	13.71	22.2	11.40	3.54	23.3	1449
6000	1.930	4.53	10.19	18.74	13.09	3.96	6.14	24.3	1516
6500	3.35	6.98	14.50	15.39	5.49	5.92	12.99	25.4	1580
7000	4.69	9.97	15.69	8.32	3.28	10.95	26.9	26.4	1642
7500	6.31	12.48	12.24	5.92	8.62	19.97	51.9	27.3	1701
8000	8.21	13.19	7.80	3.42	13.99	34.7	84.8	28.2	1759
8500	9.86	11.55	5.10	9.72	22.1	56.3	107.6	29.1	1814
9000	10.90	8.79	3.26	14.03	34.1	78.8	93.8	30.0	1868
9500	10.88	6.38	6.66	20.3	49.1	92.6	54.9	30.8	1921
10000	9.87	4.07	10.95	28.2	64.8	84.8	25.6	31.6	1972
10500	8.33	6.50	14.60	37.6	76.6	59.9	11.45	32.4	2020
11000	6.84	8.47	19.17	48.3	78.6	35.2	5.81	33.2	2070
11500	5.59	10.48	24.5	58.2	68.6	19.04	3.40	34.0	2120
12000	4.79	12.96	30.9	65.4	51.6	10.75	2.29	34.7	2160
12500	3.34	15.55	37.0	67.3	34.8	6.09	1.807	35.5	2210
13000	5.78	18.77	43.3	63.4	22.5	4.08	1.622	36.2	2250
13500	9.95	22.3	48.7	54.5	14.28	3.18	1.461	36.9	2300
14000	11.44	25.9	52.3	43.5	9.58	2.76	1.586	37.6	2340
14500	13.73	31.4	54.2	30.4	6.25	2.43	1.670	38.2	2380
15000	14.95	33.8	52.4	24.3	3.04	2.33	1.754	38.9	2420

TABLE VI.—TRANSPORT PROPERTIES OF AIR—Concluded

(c) Prandtl number

T, °K	Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001
500	0.738	0.738	0.738	0.738	0.738	0.738	0.738
1000	.756	.756	.756	.756	.756	.756	.756
1500	.767	.767	.767	.767	.767	.767	.767
2000	.773	.773	.773	.766	.724	.668	.614
2500	.762	.751	.696	.645	.611	.654	.771
3000	.740	.680	.627	.636	.740	.745	.714
3500	.678	.631	.660	.744	.737	.658	.606
4000	.640	.662	.762	.759	.619	.580	.587
4500	.654	.743	.752	.610	.578	.611	.764
5000	.702	.767	.611	.581	.624	.799	.993
5500	.748	.620	.583	.617	.785	.989	.871
6000	.763	.592	.602	.736	.969	.891	.455
6500	.610	.592	.673	.906	.955	.464	.392
7000	.593	.620	.796	.986	.830	.404	.361
7500	.595	.688	.927	.969	.424	.371	.342
8000	.620	.788	.983	.648	.387	.351	.322
8500	.666	.891	.943	.411	.363	.335	.279
9000	.730	.961	.807	.382	.348	.316	.200
9500	.806	.966	.497	.364	.336	.279	.114
10000	.886	.872	.429	.348	.319	.216	.0576
10500	.937	.532	.404	.339	.295	.145	.0314
11000	.955	.463	.382	.327	.254	.0877	.0213
11500	.947	.434	.369	.312	.201	.0524	.0167
12000	.908	.412	.355	.292	.146	.0346	.0143
12500	.728	.396	.343	.263	.101	.0238	.0129
13000	.525	.383	.333	.227	.0688	.0190	.0121
13500	.438	.369	.319	.185	.0470	.0162	.0110
14000	.421	.360	.302	.144	.0345	.0149	.0108
14500	.401	.349	.277	.0986	.0245	.0130	.0109
15000	.394	.341	.253	.0819	.0129	.0120	.0110

TABLE VII.—PARTIAL COEFFICIENTS

T, °K	(a) Specific heat, $\dfrac{ZC_{p'}}{R} = Z\sum x_i\left(\dfrac{C_i}{R}+1\right)$							(b) Thermal conductivity ratio, $\dfrac{k_n}{k_0}$						
	Pressure, atmospheres							Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001	100	10	1.0	0.1	0.01	0.001	0.0001
500	3.59	3.59	3.59	3.59	3.59	3.59	3.59	1.021	1.021	1.021	1.021	1.021	1.021	1.021
1000	3.96	3.96	3.96	3.96	3.96	3.96	3.96	1.100	1.100	1.100	1.100	1.100	1.100	1.100
1500	4.20	4.20	4.20	4.20	4.20	4.20	4.20	1.150	1.150	1.150	1.150	1.150	1.150	1.150
2000	4.33	4.33	4.33	4.33	4.33	4.33	4.34	1.177	1.177	1.177	1.177	1.177	1.178	1.181
2500	4.41	4.41	4.41	4.41	4.42	4.45	4.47	1.195	1.195	1.196	1.198	1.206	1.225	1.252
3000	4.46	4.47	4.47	4.48	4.51	4.52	4.53	1.205	1.208	1.215	1.235	1.268	1.285	1.289
3500	4.50	4.52	4.52	4.54	4.56	4.55	4.56	1.218	1.230	1.259	1.289	1.310	1.314	1.320
4000	4.53	4.55	4.57	4.58	4.58	4.60	4.65	1.238	1.270	1.312	1.329	1.340	1.356	1.406
4500	4.57	4.59	4.60	4.61	4.64	4.74	4.97	1.269	1.319	1.346	1.366	1.396	1.482	1.674
5000	4.60	4.61	4.64	4.68	4.81	5.12	5.46	1.311	1.357	1.389	1.428	1.541	1.777	1.983
5500	4.62	4.66	4.71	4.85	5.20	5.62	5.76	1.351	1.406	1.448	1.569	1.829	2.08	2.14
6000	4.64	4.71	4.85	5.20	5.73	5.94	6.00	1.383	1.457	1.506	1.824	2.14	2.25	2.50
6500	4.72	4.82	5.12	5.71	6.11	6.21	6.25	1.459	1.543	1.767	2.14	2.33	2.69	2.67
7000	4.78	5.00	5.53	6.16	6.35	6.44	6.56	1.519	1.685	2.05	2.38	2.47	2.86	2.79
7500	4.89	5.28	6.02	6.40	6.58	6.68	6.97	1.606	1.889	2.33	2.52	3.14	3.04	2.90
8000	5.04	5.68	6.42	6.66	6.78	6.97	7.57	1.733	2.14	2.55	2.66	3.33	3.11	2.75
8500	5.27	6.06	6.66	6.91	6.97	7.34	8.41	1.898	2.40	2.70	3.64	3.47	3.29	2.29
9000	5.55	6.42	6.87	6.98	7.20	7.85	9.35	2.10	2.61	2.83	3.85	3.56	2.95	1.771
9500	5.85	6.66	6.98	7.11	7.49	8.54	10.11	2.32	2.78	2.96	4.03	3.62	2.66	1.307
10000	6.15	6.89	7.07	7.26	7.86	9.29	10.52	2.53	2.93	3.03	4.21	3.54	2.23	1.105
10500	6.41	7.02	7.12	7.45	8.34	9.95	10.73	2.71	3.15	3.04	4.34	3.38	1.883	1.061
11000	6.59	7.05	7.20	7.67	8.91	10.41	10.84	2.86	3.18	3.06	4.41	3.10	1.643	1.092
11500	6.71	7.06	7.33	7.97	9.49	10.67	10.91	2.97	3.24	3.02	4.39	2.84	1.582	1.152
12000	6.76	7.07	7.39	8.33	9.99	10.82	10.94	3.06	3.29	2.99	4.39	2.59	1.573	1.223
12500	6.88	7.10	7.54	8.75	10.37	10.90	10.98	3.17	3.26	2.88	4.28	2.44	1.643	1.301
13000	6.86	7.11	7.71	9.20	10.62	10.96	10.99	3.37	3.27	2.76	4.13	2.42	1.702	1.381
13500	6.91	7.17	7.95	9.63	10.77	10.99	11.01	3.51	3.27	2.58	3.97	2.43	1.814	1.461
14000	6.90	7.23	8.24	10.01	10.87	11.00	11.02	3.55	3.20	2.41	3.90	2.49	1.892	1.586
14500	6.90	7.37	8.66	10.37	10.94	11.01	11.02	3.56	3.12	2.11	3.79	2.56	2.01	1.670
15000	6.88	7.44	8.88	10.52	11.02	11.02	11.03	3.59	3.09	1.930	3.84	2.64	2.13	1.754

TABLE VII.—PARTIAL COEFFICIENTS—Concluded

T,°K	(c) Prandtl number, Pr'							(d) Lewis number, Le'						
	Pressure, atmospheres							Pressure, atmospheres						
	100	10	1.0	0.1	0.01	0.001	0.0001	100	10	1.0	0.1	0.01	0.001	0.0001
2,000				0.773	0.772	0.772	0.772				1.355	1.355	1.346	1.335
2,500	0.775	0.775	0.774	.773	.770	.763	.750	1.408	1.407	1.404	1.391	1.355	1.233	1.172
3,000	.777	.777	.773	.762	.747	.739	.738	1.445	1.437	1.405	1.327	1.213	1.157	1.147
3,500	.776	.772	.756	.744	.738	.734	.734	1.464	1.424	1.320	1.227	1.167	1.154	1.143
4,000	.771	.758	.743	.738	.726	.729	.717	1.449	1.347	1.225	1.180	1.144	1.130	1.051
4,500	.764	.747	.739	.732	.725	.708	.683	1.404	1.265	1.196	1.160	1.114	.998	.786
5,000	.753	.739	.732	.723	.704	.682	.683	1.327	1.215	1.158	1.105	.970	.750	.609
5,500	.744	.732	.724	.705	.684	.686	.694	1.268	1.167	1.115	.982	.762	.610	.580
6,000	.740	.724	.709	.687	.691	.697		1.235	1.131	1.019	.808	.628	.578	
6,500	.728	.715	.693	.689	.703			1.158	1.071	.885	.669	.588		
7,000	.720	.700	.684	.703	.711			1.118	.975	.746	.605	.574		
7,500	.711	.690	.696	.710				1.059	.860	.652	.587			
8,000	.698	.690	.710	.721				.979	.756	.601	.570			
8,500	.691	.693	.719					.893	.669	.570				
9,000	.687	.707	.734					.805	.618	.567				
9,500	.687	.714						.726	.584					
10,000	.695	.725						.662	.558					
10,500	.704							.618						
11,000	.710							.585						
11,500	.716							.564						
12,000	.719							.531						
12,500	.719							.525						

TABLE VIII.—CONVERSION FACTORS

Multiply	By	To get	In
$\dfrac{ZE}{RT}$ or $\dfrac{ZH}{RT}$	0.0686 T (°K) 0.287 T (°K) 0.0686 T (°R) 1716 T (°R)	Energy or enthalpy	cal/gm joule/gm Btu/lb mass ft-lb/slug
$\dfrac{ZS}{R}$, $\dfrac{ZC_v}{R}$, $\dfrac{ZC_p}{R}$, or $\dfrac{ZC_p'}{R}$	0.0686 0.287 0.0686 1716	Entropy or specific heat	cal/gm °K joule/gm °K Btu/lb mass °R ft-lb/slug °R
η/η_0	$1.462\times10^{-5}\,T^{1/2}\left(1+\dfrac{112}{T}\right)^{-1}$ (T in °K) $2.28\times10^{-8}\,T^{1/2}\left(1+\dfrac{202}{T}\right)^{-1}$ (T in °R)	Coefficient of viscosity	gm/cm-sec lb sec/ft²
$\dfrac{k}{k_0}$ or $\dfrac{k_n}{k_0}$	$4.76\times10^{-6}\,T^{1/2}\left(1+\dfrac{112}{T}\right)^{-1}$ (T in °K) $1.994\times10^{-5}\,T^{1/2}\left(1+\dfrac{112}{T}\right)^{-1}$ (T in °K) $2.39\times10^{-7}\,T^{1/2}\left(1+\dfrac{202}{T}\right)^{-1}$ (T in °R) $1.856\times10^{-4}\,T^{1/2}\left(1+\dfrac{202}{T}\right)^{-1}$ (T in °R)	Coefficient of thermal conductivity	$\dfrac{cal}{cm\text{-}sec\ °K}$ $\dfrac{watt}{cm\ °K}$ $\dfrac{Btu}{ft\text{-}sec\ °R}$ $\dfrac{ft\text{-}lb}{ft\text{-}sec\ °R}$